Vibration Attenuation Based on Model Prediction Control and Equivalent-Input-Disturbance

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Abstract. This paper presents a vibration attenuation method for mechanical systems with elastic transmission parts based on the combination of the model predictive control (MPC) and the equivalent-input-disturbance approach (EID). We first design a quadratic-programming based MPC method to track the reference input and attenuate the vibration. Then we use the EID approach to attenuate external disturbances. Simulation results demonstrate the effectiveness of our method.

Keywords: Vibration attenuation · Model-predictive control · Equivalent-input-disturbance.

1 Introduction

Harmonic gears, speed reduction belts, flexible links, and other elastic connections in servo systems cause motion transmission errors. When a servo system stops suddenly during high-speed operation, it usually generates vibration at the load end. This not only degrades the positioning performance of a servo system but may also lead to system instability. Therefore, vibration suppression is of great significance in high-precision motion control.

Model prediction control (MPC) [1] is an optimization-based control law that computes optimal control actions by considering current and future constraints. This optimization ensures stability through a bimodal paradigm and enforces operational constraints through recursive feasibility constraints. Optimal performance matched with constraint feasibility and stability guarantees is a positive addition to any control system, and this is no exception in active vibration attenuation systems.

External disturbances are an important factor affecting servo systems [2]. If a system fails to process them, they will severely affect the MPC performance and degrade the control accuracy.

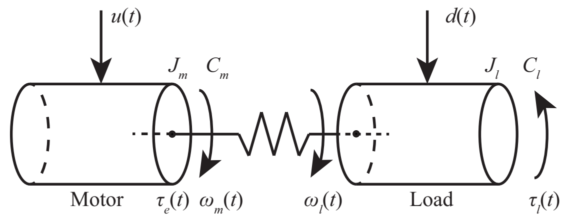

Fig. 1. Two-inertia mechanical transmission model.

Active-disturbance-rejection methods utilize observers to actively estimate and compensate for disturbances. The equivalent-input-disturbance (EID) [3] approach is one of those methods. An EID is a signal on the control input channel of a plant that produces the same effect on the output as actual disturbances do. It achieves satisfactory disturbance attenuation performance in servo systems [4], buildings vibration depression [5], and many other applications.

This paper solved the problem of vibration attenuation of mechanical systems with elastic connection devices based on the MPC and the EID approach.

2 Vibration analysis of two-inertia mechanical transmission systems

A typical two-inertia mechanical transmission system (Figure. 1) consists of a motor, a load coupled, and an elastic transmission shaft. When the transmission shaft is deformed, a torsional torque is generated. This torque is regarded as the load torque of the motor and the driving torque of the load.

2.1 Mathematical model of two-inertia mechanical transmission systems

The following equations describe the two-inertia mechanical transmission system system

$$\begin{cases} J_m \ddot{\theta}_m(t) = \tau_e(t) - \tau_w(t) \\ J_l \ddot{\theta}_l(t) = \tau_w(t) \\ \tau_e(t) = \dfrac{K_t}{R}\left[u(t) - K_e \omega_m(t)\right] \\ \tau_w(t) = C_w \left[\dot{\theta}_m(t) - \dot{\theta}_l(t)\right] + K\left[\theta_m(t) - \theta_l(t)\right] \\ \omega_m(t) = \dot{\theta}_m(t) \\ \omega_l(t) = \dot{\theta}_l(t). \end{cases} \tag{1}$$

All the parameters are summarized in Table. 1.

Table 1. System Parameters

Parameter	**Explanation**
C_w	Damping coefficient
J_m	Motor inertia
J_l	Load inertia
ω_m	Motor speed
ω_l	Load speed
θ_m	Motor position
θ_l	Load position
τ_m	Electromagnetic torque
τ_l	Load torque
τ_w	Shaft torque
K	Shaft stiffness coefficient

We choose the state variations

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \omega_m \\ \omega_l \\ \theta_m \\ \theta_l \end{bmatrix}. \tag{2}$$

The output $y = x_4$. Then, the state space formula is

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} &= \begin{bmatrix} -\frac{K_e K_t}{J_m R} - \frac{C_w}{J_m} & \frac{C_w}{J_m} & -\frac{K}{J_m} & \frac{K}{J_m} \\ \frac{C_w}{J_l} & -\frac{C_w}{J_l} & \frac{K}{J_l} & -\frac{K}{J_l} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \frac{K_t}{J_m R} & 0 \\ 0 & -\frac{1}{J_l} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ \tau_l \end{bmatrix} \\ y &= \begin{bmatrix} 0 \; 0 \; 0 \; 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}. \end{aligned} \tag{3}$$

2.2 Vibration analysis

From the analysis above, the transfer function from electromagnetic torque τ_e to motor speed ω_m is

$$G_1(s) = \frac{\omega_m(s)}{T_e(s)} = \frac{J_l s^2 + C_w s + K}{J_m J_l s^3 + (J_m + J_l) K s}. \tag{4}$$

The transfer function from electromagnetic torque τ_m to load speed ω_l is

$$G_2(s) = \frac{\omega_l(s)}{T_e(s)} = \frac{C_w s + K}{J_m J_l s^3 + (J_m + J_l) K s}. \tag{5}$$

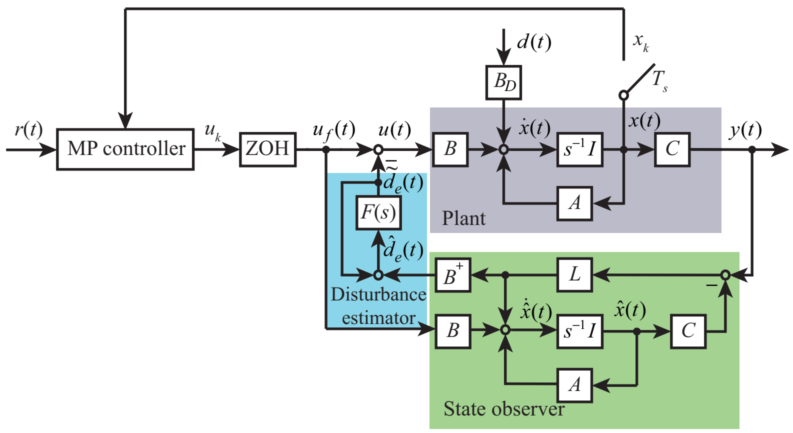

Fig. 2. System configuration

Then, the transfer function from motor speed ω_m to load speed ω_l is

$$G_3(s) = \frac{\omega_l(s)}{\omega_m(s)} = \frac{C_w s + K}{J_l s^2 + K}. \tag{6}$$

From (6), the transfer function is a second-order oscillation element. If the system is stable, the output of the load is an oscillating curve. The natural frequency and damping ratio is

$$\omega_n = \sqrt{\frac{K}{J_L}}, \zeta = \sqrt{\frac{C_w^2}{4J_L K}}. \tag{7}$$

3 Control System Configuration

In this section, we designed the control system by combining MPC and EID (Fig. 2). We use the MPC method to track the reference input and attenuate the vibration. Then, we use the EID approach to estimate and compensate for the disturbance.

3.1 Quadratic programming-based MPC

MPC is an advanced control method that minimizes a cost function containing future error vectors. These errors are evaluated as the difference between the desired setpoint trajectory and the predicted dynamic behavior of the system to be controlled. The controller evaluates the predicted dynamic behavior using a system model, which can be obtained analytically or empirically through dynamic testing. The general MPC architecture is shown in Fig. 3.

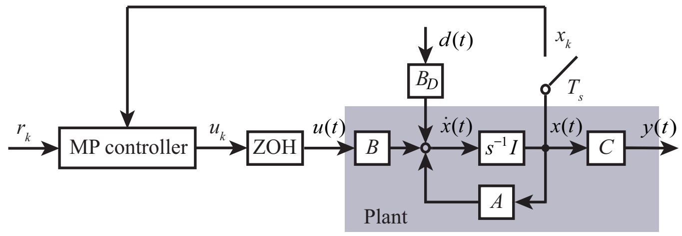

Fig. 3. MPC structure.

Consider a system described by an linear and time-invariant state-space model:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_D d(t) \\ y(t) = Cx(t). \end{cases} \tag{8}$$

The corresponding discrete model is

$$\begin{aligned} x_{k+1} &= A_d x_k + B_d u_k + B_{Dd} d_k, \\ y_k &= C_d x_k, \end{aligned} \tag{9}$$

where x_k is a state vector, u_k is an input vector and y_k is an output vector. Matrices A_d, B_d, and C_d are the state transition matrix, input and output matrix. The integer k denotes sampling instances. This system is subject to the following constraints:

$$\overline{u} \le u_k \le \underline{u}, \tag{10}$$

where the under and over lines denote lower and upper bound vectors.

Assume that the disturbance $d_k = 0$ and choose a prediction horizon N_c, the predicted state and output are given as follows

$$\begin{aligned} X =& F X_0 + \Phi U, \\ Y =& F_y X_0 + \Phi_y U, \end{aligned} \tag{11}$$

where

$$\begin{cases} X = [x_{k+1}, x_{k+2}, \dots, x_{k+N_c}]^{\mathrm{T}} \\ Y = [y_{k+1}, y_{k+2}, \dots, y_{k+N_c}]^{\mathrm{T}} \\ X_0 = x_k \\ U = [u_k, u_{k+1}, \dots, u_{k+N_c-1}]^{\mathrm{T}} \\ F = \left[A_d, A_d^2, \cdots, A_d^{N_c}\right]^{\mathrm{T}} \\ \varPhi = \begin{bmatrix} B_d & 0 & \cdots & 0 \\ A_d B_d & B_d & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_d^{N_c-1} B_d & A_d^{N_c-2} B_d & \cdots & B_d \end{bmatrix} \\ F_y = C_d F \\ \varPhi_y = C_d \varPhi. \end{cases} \tag{12}$$

To find the solution to the MPC problem, perform the following algorithm at each sampling instant.

MPC algorithm:

Step 1) Observe or measure actual system state x_k at sample k.
Step 2) Minimize the cost function

$$J(Y,U) = (Y - Y_{ref})^{\mathrm{T}} Q_e (Y - Y_{ref}) + U^{\mathrm{T}} R_e U. \tag{13}$$

The cost function can be rewritten as

$$J(Y,U) = U^{\mathrm{T}} H U + 2U^{\mathrm{T}} G + P, \tag{14}$$

where

$$\begin{aligned} H &= \varPhi_y^{\mathrm{T}} Q_e \varPhi_y + R_e, \\ G &= \varPhi_y^{\mathrm{T}} Q_e M, \\ P &= M^{\mathrm{T}} Q_e M, \\ M &= F_y X_0 - Y_{ref}, \end{aligned} \tag{15}$$

where Q_e is the state weighting matrix, and R_e is the input weighting matrix. Both Q_e and R_e are symmetric matrices.

Step 3) Apply the first element of the optimal control input vector U moves to the controlled system, and go to Step 1).

It is clear that finding a solution to MPC is converted to solving a quadratic programming (QP) problem:

$$\begin{aligned} \min: J &= \frac{1}{2} U^{\mathrm{T}} H U + U^{\mathrm{T}} \\ \text{s.t. } &\underline{u} \le U \le \overline{u} \end{aligned} \tag{16}$$

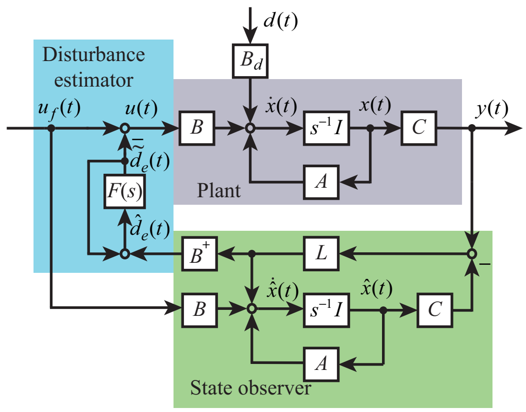

Fig. 4. Basic configuration of EID approach.

3.2 EID configuration

As mentioned above, we assume that the disturbance $d_k = 0$. However, disturbances widely exist in practical control systems and their exact value is usually not available. Disturbances severely degrade the MPC performance. Thus, we integrated the EID approach to attenuate the disturbance.

The EID approach is a kind of active disturbance rejection method. The structure is simple and its disturbance attenuation performance is satisfactory. It considers a linear plant with a disturbance (8). It can be interpreted as a plant with an EID, $d_e(t)$. That is,

$$\begin{cases} \dot{x}\left(t\right) = Ax\left(t\right) + B\left[u\left(t\right) + d_e\left(t\right)\right] \\ y\left(t\right) = Cx\left(t\right) \end{cases}. \tag{17}$$

She et al. devised a method to produce an approximation of $d_e(t)$ in a simple way (Fig. 4). The EID is estimated by a full-order observer

$$\begin{cases} \dot{\hat{x}}\left(t\right) = A\hat{x}\left(t\right) + Bu_f\left(t\right) + L\left[\hat{y}\left(t\right) - C\hat{x}\left(t\right)\right] \\ \hat{y}\left(t\right) = C\hat{x}\left(t\right) \end{cases} \tag{18}$$

where $\hat{x}\left(t\right)$ and $\hat{y}\left(t\right)$ are the reconstructed states of $x\left(t\right)$ and $y\left(t\right)$, L is the observer gain, and $u_f(t)$ is the control input of the observer. An estimate of the EID is

$$\hat{d}(t) = B^+ LC\Delta x(t) + u_f(t) - u(t) \tag{19}$$

where

$$\Delta x(t) = x(t) - \hat{x}(t) \tag{20}$$

and

$$B^+ = \left(B^T B\right)^{-1} B^T. \tag{21}$$

Table 2. System Specifications

Parameter	Value
C_w	0.0500 $\mathrm{N \cdot m/(rad/s)}$
J_m	0.0070 $\mathrm{kg \cdot m^2}$
J_l	0.0050 $\mathrm{kg \cdot m^2}$
R	0.9580 Ω
K_e	1.5000 $\mathrm{V/(rad/s)}$
K_t	1.5000 $\mathrm{N \cdot m/A}$
K	5.0000 $\mathrm{N/rad}$

A low-pass filter $F(s)$ is used to select the estimation frequency band. $\tilde{d}(t)$ is the filtered signal of $\hat{d}(t)$:

$$\tilde{d}(t) = F(t)\hat{d}(t). \tag{22}$$

4 Parameter Design

Substituting the system variables (Table 2) into the state-space formula (3), we get the matrices A, B, and C

$$A = \begin{bmatrix} -340 & 4.28 & -571 & 571 \\ 2 & -2 & 267 & -267 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \; B = \begin{bmatrix} 223 \\ 0 \\ 0 \\ 0 \end{bmatrix} \; C = \begin{bmatrix} 0 \; 0 \; 0 \; 1 \end{bmatrix}. \tag{23}$$

For the MPC, we use Euler's formula to discretize the state space formulation. That is

$$A_d = I + A \cdot T_s,\; B_d = B \cdot T_s,\; B_D d = B_D \cdot T_s,\; C_d = C. \tag{24}$$

We set the following parameters for the MPC:

$$N_c = 100,\; Q_e = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix},\; R_e = \mathbf{0}_{N_c \times N_c}. \tag{25}$$

For EID design, there are two elements need to be selected for the EID estimation and compensation [the low-pass filter $F(s)$ and the observer gain L].

First, an appropriate low-pass filter $F(s)$ ensures disturbance attenuation performance. It is selected to be a first-order low-pass one:

$$F(s) = \frac{1}{T_F s + 1}, \tag{26}$$

where T_F is used to regulate the angular-frequency band for disturbance attenuation and is selected such that

$$T_F \leq \frac{1}{5 \sim 10} \frac{1}{\omega_r} \tag{27}$$

holds. ω_r in (27) is the highest angular frequency. Thus,

$$F(j\omega) \approx 1,\ \forall \omega \in \Omega_r,\ \Omega_r = \{\omega |\ 0 \leq \omega \leq \omega_r\}, \tag{28}$$

where Ω_r is the angular-frequency band for disturbance attenuation. The natural frequency of the system vibration is about 29 rad/s. ω_r was chosen to be 29 rad/s and the low-pass filter to be

$$F(s) = \frac{1}{0.005s + 1}. \tag{29}$$

The observer gain L is chosen to satisfy the following conditions to ensure the stability of the whole system:

1) $A - LC$ is stable;
2) $||G_L F||_\infty < 1$.

The observer gain L was calculated by line quadratic regulator (LQR) in MATLAB.

$$L = \begin{bmatrix} 3330 \\ 918.9 \\ 87.53 \\ 18.44 \end{bmatrix} \tag{30}$$

5 Numerical verification

We set up a comparative simulation verifying that the combination of MPC and EID are effective in vibration attenuation. The disturbance was given as

$$d(t) = 6 \cdot 1(t - 2) - 6 \cdot 1(t - 4)\ \mathrm{N \cdot m} \tag{31}$$

for the simulations (Fig. 5) We made a set of comparison simulations among our method, conventional MPC, and PD control. The proportional gain $K_P = 3$ and derivative gain $K_D = 50$ for the PD controller. Simulation results (Fig. 6) demonstrated the effectiveness of our method when the system was affected by external disturbances.

The MPC method effectively suppressed the vibration when disturbance did not occur (0-2 s). However, when the disturbance occurs (2-4s), it cannot deal with external disturbances. The external disturbances also generate vibrations. The EID approach eliminates the influence of disturbances and thus suppresses the vibration caused by disturbances (2-6 s).

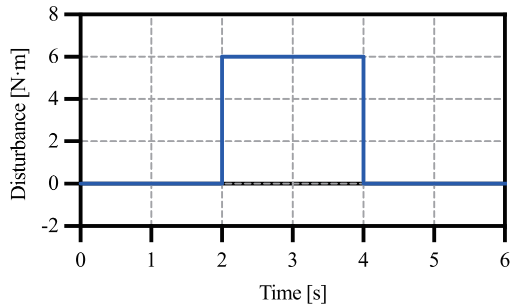

Fig. 5. Given disturbance

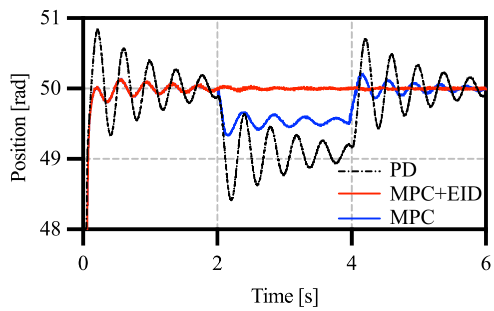

Fig. 6. Simulation results

6 Conclusion

This paper presents a vibration attenuation method based on the MPC method and the EID approach for mechanical systems with elastic transmission. This method has significant advantages:

1) The MPC-based system method precisely tracks the reference input and attenuates the vibration by solving a QP problem.
2) The EID approach effectively attenuates external disturbances and significantly improves the tracking performance of the MPC-based system.

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