Spatial incremental model-predictive repetitive control for rotational systems * **

Yujian Zhou[1,2,3], Jinhua She[4], Shijie Guo[5,6], Feng Wang[1,2,3], Seiichi Kawata[1,2,3], and Makoto Iwasaki[7]

[1] School of Automation, China University of Geosciences, Wuhan 430074, China
[2] Hubei Key Laboratory of Advanced Control and Intelligent Automation for Complex Systems, Wuhan 430074, Germany
[3] Engineering Research Center of Intelligent Technology for Geo-Exploration, Ministry of Education, Wuhan 430074, China
[4] School of Engineering, Tokyo University of Technology, Hachioji, Tokyo 192-0982
[5] Engineering Research Center of the Ministry of Education for Intelligent Rehabilitation Equipment and Detection Technologies, Hebei Key Laboratory of Robot Sensing and Human-Robot Interaction, Tianjin 300401, China
[6] School of Mechanical Engineering, Hebei University of Technology, Tianjin 300401, China
[7] Department of Electrical and Mechanical Engineering, Nagoya Institute of Technology, Nagoya 466-8555, Japan

Abstract. This paper presents a spatial repetitive control integrated with an incremental model-predictive control that achieves precise tracking of a spatial periodic reference signal for rotational systems with disturbances. We first present a modeling method in the spatial domain. This is followed by a configuration of the IMPSRC system. We designed an inner-loop feedback-linearization controller that linearizes the system model. Since the controller must contain the internal model of a periodic reference signal to achieve tracking without steady-state error, we designed an incremental model-predictive repetitive controller that contains the internal model through incremental control input. An optimization problem considers an increment of the control input between each period and ensures precise tracking of spatial periodic signals. We developed a spatial sampling method triggered by pulses of an encoder to

* This work was supported in part by the National Natural Science Foundation of China under Grants 62106240 and 52275018; the Natural Science Foundation of Hubei Province, China, under Grant 2020CFA031; China Postdoctoral Science Foundation under Grant 2022M722943; the 111 Project under Grant B17040; the "CUG Scholar" Scientific Research Funds at China University of Geosciences (Wuhan) under Project No.2022029; the Fundamental Research Funds for the Central Universities, China University of Geosciences (Wuhan) under Project No.CUG240635; and JSPS (Japan Society for the Promotion of Science) KAKENHI under Grants 22H03998 and 23K25252.
** Corresponding author: Jinhua She. E-mail: she@stf.teu.ac.jp.

implement the IMPSRC control method. The IMPSRC method was verified by simulation of tracking control of a rotational system. Simulation results show the effectiveness of the IMPSRC method.

Keywords: Spatial repetitive control · Internal principle · Incremental model-predictive control.

1 Introduction

The basic control requirement of rotational systems is to track or reject periodic signals. Repetitive control inserts the internal model of the periodic signals, for example, $1/(1-\mathrm{e}^{-Ts})$ with T being the period, and achieves perfectly asymptotic tracking [1]. However, a repetitive controller is very sensitive to the period of the signal. A small change in the period will significantly impact the control performance [2]. While most controllers are designed in the time domain, a variety of actual signals have certain periods in the spatial domain rather than in the time domain due to physical structure [3]. The period in the time domain changes as the rotational velocity changes for such signals. Thus, it is hard to ensure tracking or rejection performance with such spatial periodic signals in the time domain.

Since such signals have constant periods in the spatial domain, for example, 2π rad in a rotational system, spatial repetitive control inserts the internal model in the spatial domain to ensure the control performance for the spatial periodic signals. This concept was first used to reject an angle-dependent disturbance in a constant-speed rotational system and achieved the desired disturbance-rejection performance [4]. Since conventional robustness analysis is in the time domain, a spatial repetitive controller usually involves the synchronization between the time and the spatial domain. It is difficult to implement a spatial repetitive control method in a time-domain control system. In order to ensure synchronization and accommodate variable time periods, various methods have been proposed, including delay estimation [5], frequency alignment [6], and frequency adaptation [7]. This makes the design of a spatial method complicated and also declines the tracking performance of a repetitive controller.

Model predictive control (MPC) is a practical control method in control practices [8]. Once the internal model of a periodic signal is inserted into the controller, an MPC-based system has good control performance for rotational systems [9]. MPC methods make predictions of a number of future discrete steps. However, the internal model of a periodic signal has infinite discrete steps. This makes it challenging to insert the internal model of a periodic signal into an MPC controller. An MPC controller is designed after a simplified repetitive controller that improves current control performance for a permanent-magnet synchronous machine in [10]. An incremental state-space model of a control objective is built in [11]. An MPC controller which contains a part of control input generated by a repetitive controller is designed for high-precision tracking for a motor in linear motion. They both face a trade-off between stability and tracking precision.

This paper presents a spatial repetitive control method using an incremental model-predictive control, namely the IMPSRC method, for a mechatronic system with disturbances that ensures reference tracking performance for spatial periodic signals. First, we present a modeling method for the spatial-domain system model. Then, we design a discrete feedback-linearization controller that provides a discrete linear model. A model predictive controller calculated a control input of the system by solving an optimization problem where the internal model was considered as an incremental control input. An encoder-pulse-triggered sampling method was developed to perform a sampling task in the spatial domain. Finally, Simulation results demonstrate the validity of the IMPSRC method.

In this paper, λ is a delay operator ($= z^{-1}$), $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^+$ is the set of positive real numbers, $\mathbb{N}$ is the set of natural numbers, $\mathbb{C}$ is the set of complex numbers, $\mathbb{R}^n$ is the set of n-dimensional real vectors, and $\mathbb{R}^{m\times n}$ is the set of $m \times n$ real matrices. $\mathbb{RH}_\infty$ is the set of real rational functional in λ that have no poles within or on the unit circle, $G(\cdot)$ is a transfer function of a signal or a system. For $G(\lambda) \in \mathbb{RH}_\infty$, $\|G\|_2$ is its Euclidean norm.

2 Modeling in Spatial domain

Let $\omega(t) \in \mathbb{R}$ be a rotational velocity and let

$$\Omega(t) = |\omega(t)|. \tag{1}$$

A position is defined to be

$$\theta(t) := \int_0^t \Omega(\tau)\mathrm{d}\tau. \tag{2}$$

Definition 1 (Spatial domain). *The spatial domain is a set of positions defined by* (2) *on which mathematical functions or physical signals are defined.*

A time-domain signal, $\xi(t)$, is repsented in the spatial domain as $\bar{\xi}(\theta)$:

$$\bar{\xi}(\theta) := \bar{\xi}(\theta(t)) = \xi(t). \tag{3}$$

Consider a time-domain state-space plant

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = Ax(t) + Bu(t) + B_d d(t) \\ y(t) = Cx(t) \end{cases} \tag{4}$$

where $x(t)$ is the state, $y(t)$ is the output, $u(t)$ is the control input, and $d(t)$ is a disturbance.

Substituting (2) into (4) yields

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = \frac{\mathrm{d}x(t)}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}x(\xi(\theta))}{\mathrm{d}\theta}\Omega(\xi(\theta)) = \bar{\Omega}(\theta)\frac{\mathrm{d}\bar{x}(\theta)}{\mathrm{d}\theta}. \tag{5}$$

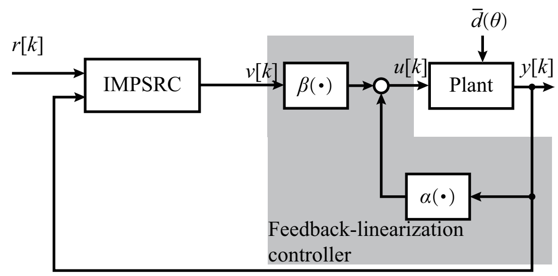

Fig. 1. Configuration of SEIDRC system.

The spatial-domain state-space model is

$$\begin{cases} \dfrac{\mathrm{d}\bar{x}(\theta)}{\mathrm{d}\theta} = \dfrac{A}{\bar{\Omega}(\theta)}\bar{x}(\theta) + \dfrac{B}{\bar{\Omega}(\theta)}\bar{u}(\theta) + B_d\dfrac{\bar{d}(\theta)}{\bar{\Omega}(\theta)} \\ \bar{y}(\theta) = C\bar{x}(\theta). \end{cases} \tag{6}$$

A spatial-domain sampling interval is

$$\theta[k+1] - \theta[k] = \Delta\theta \tag{7}$$

where $k \in \mathbb{N}$. The control input is yielded by a zero-order holder

$$u[k] = \bar{u}(k\Delta\theta),\ \ \theta \in [\theta[k], \theta[k+1]). \tag{8}$$

For a small enough $\Delta\theta$,

$$\bar{\Omega}(\theta) \approx \bar{\Omega}(\theta[k]). \tag{9}$$

A approximate description of the discrete model (6) is

$$\begin{cases} x[k+1] \approx \Gamma(\theta[k])x[k] + \Psi(\theta[k])Bu[k] \\ \qquad\qquad +B_d d_{\Omega}[k] \\ \qquad y[k] = Cx[k] \end{cases} \tag{10}$$

where

$$\Gamma(\theta[k]) = \exp\left(\frac{A}{\bar{\Omega}(\theta[k])}\Delta\theta\right) \tag{11}$$

$$\Psi(\theta[k]) = \frac{1}{\bar{\Omega}(\theta[k])}\int_0^{\Delta\theta} \exp\left(\frac{A}{\bar{\Omega}(\theta[k])}\varsigma\right)\mathrm{d}\varsigma \tag{12}$$

$$d_{\Omega}[k] = \frac{1}{\bar{\Omega}(\theta[k])}\int_{\theta[k]}^{\theta[k+1]} \exp\left(\frac{A}{\bar{\Omega}(\theta[k])}\Delta\theta\right)\bar{d}(\varsigma)\mathrm{d}\varsigma. \tag{13}$$

3 Control system design

The IMPSRC system (Fig. 1) has three parts: a plant, an exact-feedback-linearization controller, and a 2DOF repetitive controller.

The feedback-linearization controller is [12]

$$u[k] = \frac{C\Gamma(\theta[k])}{C\Psi(\theta[k])B}x[k] + \frac{1}{C\Psi(\theta[k])B}v[k] \tag{14}$$

where $v[k]$ is the visual control input of the linearized plant. Substituting (14) into (10) yields

$$y[k+1] = v[k] + d_e[k] \tag{15}$$

where $d_e[k] = CB_d d_\Omega[k] + \Delta[k]$ and $\Delta[k]$ is a model uncertainty caused by approximation (10).

Consider tracking a spatial periodic reference signal:

$$R(\lambda) = \frac{\sum_{i=0}^{N-1} r_i \lambda^i}{1-\lambda^N}. \tag{16}$$

where

$$L = \frac{2\pi}{\Delta\theta} \tag{17}$$

is the discrete period. The internal model of $R(\lambda)$ is

$$K_{\mathrm{RC}}(\lambda) = \frac{1}{1-\lambda^L}. \tag{18}$$

Thus, a discrete repetitive controller requires in the calculation

$$V(\lambda) = \lambda^L V(\lambda) + F(R(\lambda), Y(\lambda)) \tag{19}$$

where $F(\cdot,\cdot)$ is a function. Rewrite (19), there is

$$v[n] = v[n-L] + f\left(r[i], y[j]\right) \tag{20}$$

where $f\left(r[i], y[j]\right)$ is a function of reference and output signals.

Biuld a augmented model from (15) which is

$$\hat{y}[k+1] = \hat{v}[k] + \hat{d}_e[k] \tag{21}$$

where

$$\hat{y}[k+1] = \begin{bmatrix} y[k+1] \\ \vdots \\ y[k+L] \end{bmatrix}, \hat{v}[k] = \begin{bmatrix} v[k] \\ \vdots \\ v[k+L-1] \end{bmatrix}, \hat{d}_e[k] = \begin{bmatrix} d_e[k] \\ \vdots \\ d_e[k+L] \end{bmatrix} \tag{22}$$

Thus, a new augmented system model is

$$\begin{cases} \hat{x}[k+1] = \hat{x}[k] + \Delta\hat{v}[k] \\ \hat{y}[k] = \hat{x}[k] \end{cases} \tag{23}$$

where $\Delta\hat{v}[k] = \hat{v}[k] - \hat{v}[k-1]$.

An optimal control problem at k with horizon N is

Problem 1.

$$\min_{v} \quad J(r[k], y[k], v[k],) \tag{24}$$

$$\text{s.t.} \quad \begin{cases} \hat{x}[k+1|k] = \hat{x}[k|k] + \Delta\hat{v}[k|k] \\ \hat{y}[k|k] = \hat{x}[k|k] \end{cases} \tag{25}$$

where $i \in [0, N-1]$. The cost function is

$$J = \sum_{i=0}^{N-1} \left(\hat{e}^{\mathrm{T}}[k+i|k]Q\hat{e}[k+i|k] + \Delta\hat{v}^{\mathrm{T}}[k+i|k]R\Delta\hat{v}[k+i|k]\right) + \hat{e}^{\mathrm{T}}[k+N|k]P\hat{e}[k+N|k] \tag{26}$$

where $\hat{e}[k] = \hat{r}[k] - \hat{y}[k]$ and $Q \geq 0,\ R > 0,\ P \geq 0$ are weighting matrices. Note that

$$\Delta\hat{v}^2[k] = \begin{bmatrix} v[k] - v[k-L] \\ \vdots \\ v[k+L-1] - v[k-1] \end{bmatrix}. \tag{27}$$

The increment of virtual input of each adjacent spatial period, $\Delta v[k]$, is calculated to satisfy the requirement of the repetitive control for achieving tracking with a steady-state error of a periodic reference signal.

4 Feasibility and stability analysis

This section presents the stability feasibility and stability analysis of the IMP-SRC method.

Remark 1. System (21) is a stabilizible linear system.

From [13], there exists a local stabilizing controller such that by implememting the controller,it holds that

$$\hat{e}^{\mathrm{T}}[k+1]P\hat{e}[k+1] - \hat{e}^{\mathrm{T}}[k]P\hat{e}[k] + \hat{e}^{\mathrm{T}}[k]Q\hat{e}[k] + \Delta\hat{v}^{\mathrm{T}}[k]R\Delta\hat{v}[k] \leq 0. \tag{28}$$

To derive the stability of the system, a Lyapunov function is

$$V[k] = \sum_{i=0}^{k-1} \left(\hat{e}^{\mathrm{T}}[i]Q\hat{e}[i] + \Delta\hat{v}^{\mathrm{T}}[i]R\Delta\hat{v}[i]\right) + \hat{e}^{\mathrm{T}}[k]P\hat{e}[k]. \tag{29}$$

The increment of the Lyapunov function is

$$\begin{aligned} V(k+1) - V(k) &= \sum_{i=0}^{k-1} \left(\hat{e}^{\mathrm{T}}[i]Q\hat{e}[i] + \Delta\hat{v}^{\mathrm{T}}[i]R\Delta\hat{v}[i]\right) + \hat{e}^{\mathrm{T}}[k+1]P\hat{e}[k+1] \\ &\quad - \sum_{i=0}^{k-1} \left(\hat{e}^{\mathrm{T}}[i]Q\hat{e}[i] + \Delta\hat{v}^{\mathrm{T}}[i]R\Delta\hat{v}[i]\right) + \hat{e}^{\mathrm{T}}[k]P\hat{e}[k] \\ &= \hat{e}^{\mathrm{T}}[k+1]P\hat{e}[k+1] - \hat{e}^{\mathrm{T}}[k]P\hat{e}[k] \\ &\quad + \hat{e}^{\mathrm{T}}[k]Q\hat{e}[k] + \Delta\hat{v}^{\mathrm{T}}[k]R\Delta\hat{v}[k] \leq 0. \end{aligned} \tag{30}$$

Thus, the optimization problem is feasible and the system is stable.

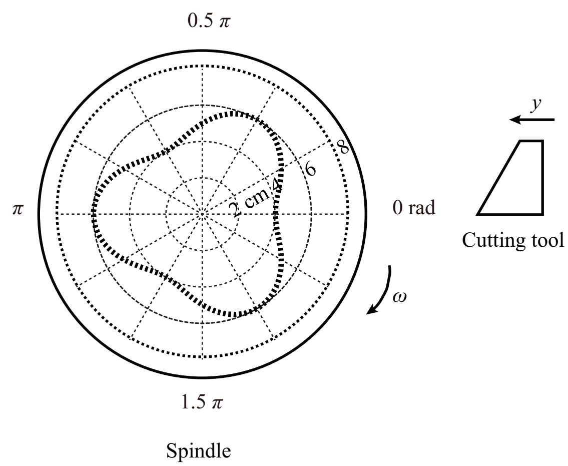

Fig. 2. Cutting process with reference trajectory (top view).

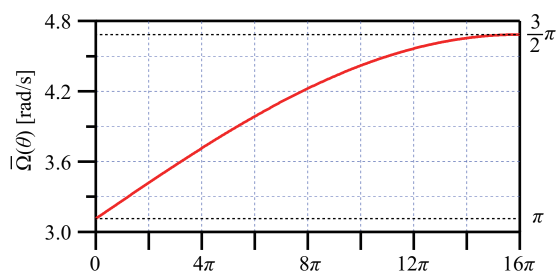

Fig. 3. Rotational velocity of the spindle.

5 Simulation verification

Consider a cutting process (Fig. 2), a workpiece rotates around a spindle and a cutting tool moves back and forth. The combination of the movements cuts the workpiece to a certain shape. We did not consider the control of the spindle and assumed that it was controlled by a tuned controller. The motor connected with the cutting tool was controlled by the IMPSRC method. The simulation was carried out using MATLAB R2022b/Simulink.

In this study, the pattern was (fig. 2)

$$\bar{r}(\theta) = 5 - \cos 3\theta \text{ cm}. \tag{31}$$

Chose the system matrices of (4) to be

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -21.12 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C = \begin{bmatrix} 657.55 & 0 \end{bmatrix}.$$

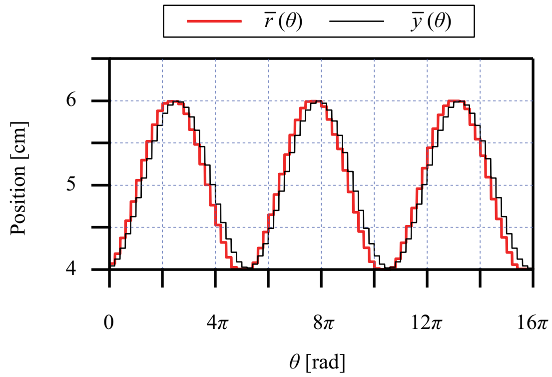

Fig. 4. Input-output characteristic after linearization.

The initial position of the cutting tool was

$$\theta(0) = 0 \text{ rad}, \ \bar{r}(0) = 4 \text{ cm}.$$

The desired rotational velocity of the spindle was (Fig. 3)

$$\bar{\Omega}(\theta) = \frac{\pi}{3} + \frac{\pi}{6}\sin\frac{\theta}{32} \text{ rad/s}. \tag{32}$$

The disturbance was chosen as

$$\bar{d}(\theta) = 0.5\sin 5\theta + 0.3\sin 3\theta \text{ V}, \quad B_d = \begin{bmatrix} 0 \ 1 \end{bmatrix}^{\mathrm{T}}. \tag{33}$$

The spatial sampling interval was chosen to be

$$\Delta\theta = \frac{2\pi}{100} \text{ rad } (= 3.6^\circ)$$

which means that a sampling occurs when 3.6° passed. We used an Incremental Shaft Encoder and triggered blocks to implement the spatial sampling. Pulses per revolution was set to be 5000. Thus, a sampling signal was generated every 500 pulses. Calculation and control were triggered when a sampling signal appeared.

To carry on the simulation of the IMPSRC method, we first verified the linearization controller. As in (14), the linearization controller was implemented using matrix exponential of system matrices and spatially sampled rotational angle. Simulation result (Fig. 4) shows that after linearization, the input-output characteristic of the system is nearly a one-step delay discrete system.

Then, we performed the IMPSRC method in simulation. We chose the perdictive horizon as $N = 3$. The weighting matrices were chosen to be

$$Q = R = P = I_{100} \tag{34}$$

where $I_{100} \in \mathbb{R}^{100\times 100}$ is a identy matrix. The results (Fig. 5) show that the system enters a steady state. The influence of the disturbance and uncertainty is suppressed to lower than 0.2 cm in the steady state.

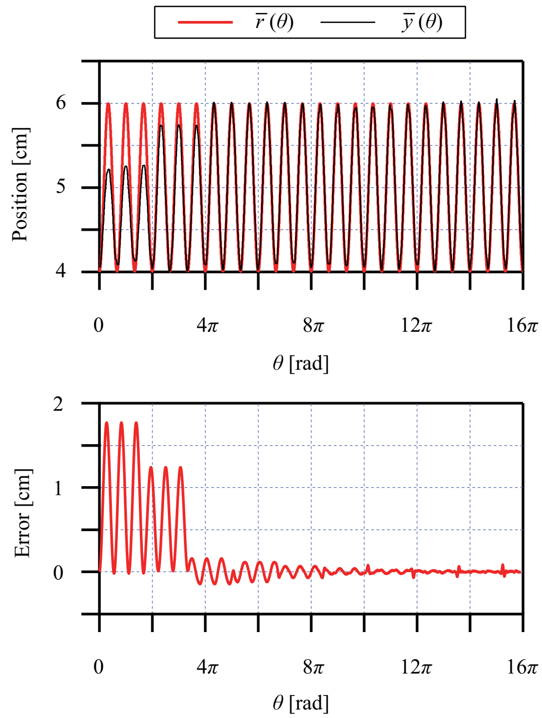

Fig. 5. Results of reference tracking with disturbance (33).

6 Conclusion

This study presents an IMPSRC method for rotational systems tracking periodic signals in the spatial domain. The designed system achieves the desired tracking performance for spatial periodic references and rejects the disturbances. First, we showed a modeling method in the spatial domain. Then, we presented the configuration of the IMPSRC system. We presented a method to perform feedback linearization for a discrete system. We showed an optimization problem based on the repetitive control that achieves tracking without steady-state error theoretically by using it in an incremental MPC control. We verified the IMPSRC method by simulations. The simulation results show that the method achieves the desired performance of both reference tracking and disturbance rejection.

References

1. Hara, S., Yamamoto, Y., Omata, T. Nakano, M.: Repetitive control system: a new type servo system for periodic exogenous signals. IEEE Transactions on Automatic Control **33**(7), 659–668 (1988)
2. Zhou, L., Gao, D., She, J.: Tracking control for a position-dependent periodic signal in a variable-speed rotational system. Automatica **158**, 111282 (2023)

3. Dini, P., Saponara, S.: Review on model based design of advanced control algorithms for cogging torque reduction in power drive systems. Energies **15**(23), 8990 (2022)
4. Nakano, M., She, J.-H., Matsuo, Y., Hino, T.: Limination of position-dependent disturbances in constant-speed-rotation control systems. Control Engineering Practice **4**(9), 1241-1248 (1996)
5. Castro, R. S., Flores, J. V., Salton, A. T.: Robust discrete-time spatial repetitive controller. IEEE Transactions on Control Systems Technology **27**(6), 2696–2702 (2019)
6. Huo, X., Wang, M., Liu, K.-Z., Tong, X,: Attenuation of position-dependent periodic disturbance for rotary machines by improved spatial repetitive control with frequency alignment. IEEE/ASME Transactions on Mechatronics **25**(1), 339–348 (2020)
7. Zhou, L., Gao, D., She, J., Zhang, X.-M., Yan, H.: Adaptive spatial repetitive-control for time-varying periodic signals in a rotational system. IEEE Transactions on Automatic Control, 1–8 (2024)
8. Wang, L.: Model predicitive control system design and implementation using MATLAB. Springer (2009)
9. Francis, B. A., Wonhan, W. M.: The internal model principle of control theory. Automatica **12**(5), 457–465 (1976)
10. Liu, Y., Cheng, S., Ning, B., Li, Y. Robust model predictive control with simplified repetitive control for electrical machine drives. IEEE Transactions on Power Electronics **34**(5), 4524–4535 (2019)
11. Cao, R., Low, K. -S. A repetitive model predictive control approach for oprecision tracking of a linear motion system. IEEE Transactions on Industrial Electronics **56**(6), 1955–1962 (2009)
12. Isidori, A.: Nonlinear control systems. Springer London (1995)
13. Xie, H., Dai, L., Lu, Y., Xia, Y. Disturbance rejection mpc framework for input-affine nonlinear systems. IEEE Transactions on Automatic Control **67**(12), 6595—6610 (2022)