Dynamic Optimization of CHP Unit Variable Load Process Based on Hybrid of Gauss Pseudospectral Method and Direct Shooting Method

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Abstract. A dynamic optimization strategy based on a hybrid of pseudospectral method and direct shooting method is proposed for the problem of fast optimization of the variable load control process of CHP units. Firstly, the dynamic optimization proposition of the optimal variable load control process of a CHP unit is constructed and discretized into a nonlinear programming problem. Then, the feasible solutions corresponding to the state variables and control variables are solved using the Gauss pseudospectral method, and the serial optimization strategy from feasible solutions to near-optimal solutions is used. Finally, the control variables are discretized at the nodes as initial values for the direct shooting method, and then the sequential quadratic programming (SQP) algorithm is used to solve the exact optimal solution. Simulations were conducted on three cases of a 300MW CHP unit, and the results showed that the proposed method reduced the calculation time by approximately 32%, 36%, and 53%, respectively.

Keywords: variable load control process, dynamic optimization, Gauss pseudospectral method, direct shooting method, optimal control.

1 Introduction

In integrated energy system, the dynamic regulation characteristics of technologically modified combined heat and power (CHP) units are complex [1], and their mechanism analysis presents nonlinear characteristics such as multivariate coupling and multiparameter superposition, which leads to the difficulty of accurate modeling and rapid simulation of the variable load response capability of CHP units. As a result, solving the optimal control problem of the transformed CHP unit efficiently and reliably has become an emerging problem for integrated energy use.

Several researches have been conducted on the CHP unit model and optimal control. In [2], a dynamic model of a heating unit in the form of differential equations of load-pressure is developed, which analyzes the object control characteristics. In [3], the optimal control method of the unit is investigated under grid-connected conditions of wind power, which uses a heating unit's DAE model. To meet the needs of CHP units to participate in scenarios such as comprehensive energy frequency regulation and scheduling, control strategies to improve the fast regulation capability of the units have been continuously proposed [4-5]. However, it is difficult to meet the computational performance requirements of the optimization of different scenarios by using the optimal control algorithm for the variable load process of CHP units.

In recent years, the pseudospectral method has become one of the most popular methods for addressing optimum control dynamic problems. In [6], the Gauss pseudospectral method is proposed to obtain higher solution accuracy with fewer nodes and faster convergence. In [7], the gliding vehicle trajectory optimization problem is studied, which uses the Gauss pseudospectral method. In [8], an optimization strategy combining particle swarm algorithms and Gauss pseudospectral method is proposed to solve the hybrid trajectory planning problem. In [9], the state-dependent in Gauss pseudospectral method for rigid spacecraft attitude control is presented. Due to the unreasonable selection of configuration points and initial values, the above methods often result in low accuracy or increased computational complexity when solving optimal control problems.

The remaining part of the paper is organized as follows. The Section 2 establishes the CHP unit's variable load control system model. The Section 3 proposes a hybrid optimization strategy to solve the dynamic optimization problem. The simulation result is presented in Section 4. Conclusions are given in Section 5.

2 Variable Load Control System Model of the CHP Unit

2.1 Mechanistic Model of CHP Unit

Through analyzing the characteristics of the thermal components of the heating unit and the steam work process in the turbine [3], a nonlinear mechanism model of the simplified CHP unit can be obtained.

Dynamic Relationship of Coal-fired Pulverizing System.

$$V_{\mathrm{m}}(t)=V_{\mathrm{B}}(t-t_{\mathrm{B}}) \tag{1}$$

$$T_{\mathrm{f}}\dot{V}_{\mathrm{f}}(t)=-V_{\mathrm{f}}(t)+V_{\mathrm{B}}(t-t_{\mathrm{B}}) \tag{2}$$

where $V_{\mathrm{B}}(t)$ is unit coal feed mass flow rate; $V_{\mathrm{m}}(t)$ is actual coal feed mass flow rate of the pulverizing system; t_{B} is delay time constant of the pulverizing process; $V_{\mathrm{f}}(t)$ is boiler combustion rate; T_{f} is pulverizing inertia time constant.

Dynamic Relationship of Drum Boiler System.

$$C_{\mathrm{d}}\dot{\psi}_{\mathrm{d}}(t)=-K_3\psi_{\mathrm{t}}(t)V_{\mathrm{T}}(t)+K_1V_{\mathrm{f}}(t) \tag{3}$$

$$\psi_{\mathrm{d}}(t)-\psi_{\mathrm{t}}(t)=K_2\left(K_1V_{\mathrm{f}}(t)\right)^{1.5} \quad (4)$$

where $V_{\mathrm{T}}(t)$ is turbine regulating gate opening; $\psi_{\mathrm{d}}(t)$ is steam package pressure; $\psi_{\mathrm{t}}(t)$ is main steam pressure; C_{d} is boiler heat storage coefficient.

Dynamic Relationship of Turbine System.

$$T_{\mathrm{t}}\dot{P}_{\mathrm{H}}(t)=K_3K_4\psi_{\mathrm{t}}(t)V_{\mathrm{T}}(t)+K_5\psi_{\mathrm{z}}(t)V_{\mathrm{H}}(t)-P_{\mathrm{H}}(t) \quad (5)$$

$$\psi_1(t)=0.01\psi_{\mathrm{t}}(t)V_{\mathrm{T}}(t) \quad (6)$$

$$C_{\mathrm{z}}\dot{\psi}_{\mathrm{z}}(t)=-K_6m_{\mathrm{r}}(t)\left(96\psi_{\mathrm{z}}(t)-\varepsilon_{\mathrm{r}}(t)+103\right)+K_3(1-K_4)\psi_{\mathrm{t}}(t)V_{\mathrm{T}}(t)-K_5\psi_{\mathrm{z}}(t)V_{\mathrm{H}}(t) \quad (7)$$

where $V_{\mathrm{H}}(t)$ is extraction regulating butterfly valve opening; $P_{\mathrm{H}}(t)$ is unit power generation; $\psi_{\mathrm{z}}(t)$ is medium pressure cylinder discharge pressure; $\psi_1(t)$ is turbine first-stage pressure; $m_{\mathrm{r}}(t)$ is circulating water mass flow rate; $\varepsilon_{\mathrm{r}}(t)$ is circulating water return temperature; T_{t} is turbine inertia time constant; C_{z} is heat storage coefficient of the heat network heater.

Dynamic Relationship of Heating Systems.

$$m_{\mathrm{H}}(t)=K_7K_6m_{\mathrm{r}}(t)\left(96\psi_{\mathrm{z}}(t)-\varepsilon_{\mathrm{r}}(t)+103\right) \quad (8)$$

where $m_{\mathrm{H}}(t)$ is unit heating extraction flow; K_1, K_2, K_3, K_4, K_5, K_6 and K_7 are static parameters.

2.2 Model of Multivariate Coordinated Control

Aiming at the above mechanism model, an analysis is conducted on the optimal control requirements of CHP units under various operating conditions and varying load. The direction and magnitude of the control variables $V_{\mathrm{B}}(t)$, $V_{\mathrm{T}}(t)$, and $V_{\mathrm{H}}(t)$ in the CHP unit are synergistically adjusted by the controllers so that the output variables $\psi_{\mathrm{t}}(t)$, $m_{\mathrm{H}}(t)$, and $P_{\mathrm{H}}(t)$ are varied accordingly. This paper designs a thermal-electric coordinated control algorithm to fulfill the three important tasks of accurate electrical power tracking, rapid thermal power recovery, and system pressure safety [2-3]. Its mathematical model can be expressed as:

$$\begin{cases} V_{\mathrm{T}}(t) = K_{\mathrm{PT}} E_{\mathrm{T}}(t) + K_{\mathrm{IT}} \int_0^t E_{\mathrm{T}}(t) dt, & E_{\mathrm{T}}(t) = \psi_{\mathrm{SP}} - \psi_{\mathrm{t}}(t) \\ V_{\mathrm{B}}(t) = K_{\mathrm{PB}} E_{\mathrm{B}}(t) + K_{\mathrm{IB}} \int_0^t E_{\mathrm{B}}(t) dt, & E_{\mathrm{B}}(t) = P_{\mathrm{SP}} - \left(m_{\mathrm{H}}(t) - \frac{K e^{-t/T_{\mathrm{c}}}}{T_{\mathrm{c}}} P_{\mathrm{H}}(t) \right) \\ V_{\mathrm{H}}(t) = K_{\mathrm{PH}} E_{\mathrm{H}}(t) + K_{\mathrm{IH}} \int_0^t E_{\mathrm{H}}(t) dt, & E_{\mathrm{H}}(t) = m_{\mathrm{SP}} - m_{\mathrm{H}}(t) \end{cases} \tag{9}$$

where K_{PT}, K_{IT}, K_{PB}, K_{IB}, K_{PH}, K_{IH}, K and T_{c} are the control system regulatory parameters. ψ_{SP}, P_{SP} and m_{SP} represent main steam pressure, electric power and heat supply extraction flow settings, respectively.

2.3 Output Variable Constraints of Control Process

Considering the safety and stability of the operation of the CHP unit, the fluctuation range of the output variable process value and the error range of the steady-state value in the control process cannot exceed the allowable limit, and the following constraints should be met:

$$\begin{cases} -\Delta_{\psi} \le \psi_{\mathrm{SP}} - \psi_{\mathrm{t}}(t) \le \Delta_{\psi} \\ -\Delta_{\mathrm{P}} \le P_{\mathrm{SP}} - P_{\mathrm{H}}(t) \le \Delta_{\mathrm{P}} \\ -\Delta_{\mathrm{m}} \le m_{\mathrm{SP}} - m_{\mathrm{H}}(t) \le \Delta_{\mathrm{m}} \end{cases} \tag{10}$$

$$\begin{cases} -\delta_{\psi} \le \psi_{\mathrm{t}}(t_{\mathrm{f}}) - \psi_{\mathrm{SP}} \le \delta_{\psi} \\ -\delta_{\mathrm{P}} \le P_{\mathrm{H}}(t_{\mathrm{f}}) - P_{\mathrm{SP}} \le \delta_{\mathrm{P}} \\ -\delta_{\mathrm{m}} \le m_{\mathrm{H}}(t_{\mathrm{f}}) - m_{\mathrm{SP}} \le \delta_{\mathrm{m}} \end{cases} \tag{11}$$

where Δ_{ψ}, Δ_{P}, and Δ_{m} represent fluctuation ranges of the main steam pressure, the electric power and the heat supply extraction flow, respectively. δ_{ψ}, δ_{P}, and δ_{m} represent error ranges of the main steam pressure, the electric power, and the heat supply extraction flow, respectively. t_0 and t_{f} represent the starting and final of the optimal control process, respectively.

3 Hybrid Optimization Strategies of Dynamic Optimization Problems

3.1 Optimization Proposition Construction

Performance Objective Function. To ensure the accurate, fast and stable variable load regulation in the control process of CHP units, the mathematical form of the combination of the control variable and output variable is selected as the objective function of optimal control performance, which is described as follows:

$$\min_{u,y} J=\int_{t_0}^{t_f}\left(\left\|\boldsymbol{u}(t)-\boldsymbol{u}(t-1)\right\|_{\boldsymbol{R}}^2+\left\|\boldsymbol{y}(t)-\boldsymbol{y}^*\right\|_{\boldsymbol{S}}^2\right)^2 dt \tag{12}$$

where $\boldsymbol{y}^*$ is the set value of the output variable; $\boldsymbol{y}(t)$ is the measured output of the control object at the $t-1$-th moment as the feedback of the closed-loop system at the t-th moment; $\|\boldsymbol{u}(t)-\boldsymbol{u}(t-1)\|_{\boldsymbol{R}}^2$ minimizes the smooth change of the control variable; R and S are positive definite weighting matrices. In the model, $\boldsymbol{u}$ is the control variable, $\boldsymbol{u}=[V_{\mathrm{T}},V_{\mathrm{B}},V_{\mathrm{H}}]^T$; $\boldsymbol{y}$ is the output variable, $\boldsymbol{y}=[\psi_{\mathrm{t}},P_{\mathrm{H}},m_{\mathrm{H}}]^T$.

Model Constraints. The model constraints are the CHP unit mechanism model, the multivariate coordinated control model, and the control process output variable constraints, as described in Sections 2.1 to 2.3.

Boundary Constraints. In the time period $[t_0,t_{\mathrm{f}}]$, the CHP unit completes the variable load regulation. The initial and termination states of each control variable and output variable are presented as follows:

$$\begin{cases}\psi_{\mathrm{t}}(t_0)=\psi_{\mathrm{t}}^0, & V_{\mathrm{T}}(t_0)=V_{\mathrm{T}}^0\\ P_{\mathrm{H}}(t_0)=P_{\mathrm{H}}^0, & V_{\mathrm{B}}(t_0)=V_{\mathrm{B}}^0\\ m_{\mathrm{H}}(t_0)=m_{\mathrm{H}}^0, & V_{\mathrm{H}}(t_0)=V_{\mathrm{H}}^0\end{cases} \tag{13}$$

$$\begin{cases}\psi_{\mathrm{t}}(t_{\mathrm{f}})=\psi_{\mathrm{t}}^{\mathrm{f}}, & V_{\mathrm{T}}(t_{\mathrm{f}})=V_{\mathrm{T}}^{\mathrm{f}}\\ P_{\mathrm{H}}(t_{\mathrm{f}})=P_{\mathrm{H}}^{\mathrm{f}}, & V_{\mathrm{B}}(t_{\mathrm{f}})=V_{\mathrm{B}}^{\mathrm{f}}\\ m_{\mathrm{H}}(t_{\mathrm{f}})=m_{\mathrm{H}}^{\mathrm{f}}, & V_{\mathrm{H}}(t_{\mathrm{f}})=V_{\mathrm{H}}^{\mathrm{f}}\end{cases} \tag{14}$$

where $(\psi_{\mathrm{t}}^0,P_{\mathrm{H}}^0,m_{\mathrm{H}}^0)$ and $(\psi_{\mathrm{t}}^{\mathrm{f}},P_{\mathrm{H}}^{\mathrm{f}},m_{\mathrm{H}}^{\mathrm{f}})$ are the output values at the start and end times, respectively; $(V_{\mathrm{T}}^0,V_{\mathrm{B}}^0,V_{\mathrm{H}}^0)$ and $(V_{\mathrm{T}}^{\mathrm{f}},V_{\mathrm{B}}^{\mathrm{f}},V_{\mathrm{H}}^{\mathrm{f}})$ are the control values at the start and end times, respectively.

Path Constraints. Due to the constraints of the CHP unit's component attributes, V_{T}, V_{B}, and V_{H} must meet the following constraints in the variable load regulation process.

$$\begin{cases}V_{\mathrm{T}}^{\min}\le V_{\mathrm{T}}(t)\le V_{\mathrm{T}}^{\max}\\ V_{\mathrm{B}}^{\min}\le V_{\mathrm{B}}(t)\le V_{\mathrm{B}}^{\max}\\ V_{\mathrm{H}}^{\min}\le V_{\mathrm{H}}(t)\le V_{\mathrm{H}}^{\max}\end{cases} \tag{15}$$

where $V_{\mathrm{T}}^{\min}$ and $V_{\mathrm{T}}^{\max}$ are the minimum and maximum values of the variation of the turbine regulator opening, respectively; the definitions of variables in V_{B} and V_{H} are the same as those in V_{T}.

The CHP unit variable load optimal control problem can be stated as a dynamic optimization problem in terms of DAEs as follows:

$$\begin{cases} \min\ J\left(\boldsymbol{x}(t_{\mathrm{f}})\right) \\ s.t.\ \ \dot{\boldsymbol{x}}(t) = f\left(\boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{u}(t)\right) \\ \quad g_{\mathrm{E}}\left(\boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{u}(t)\right)=0 \\ \quad h_{\mathrm{E}}\left(\boldsymbol{x}(t_{\mathrm{f}}), \boldsymbol{y}(t_{\mathrm{f}}), \boldsymbol{u}(t_{\mathrm{f}})\right)=0 \\ \quad g_{\mathrm{IE}}\left(\boldsymbol{x}(t), \boldsymbol{y}(t), \boldsymbol{u}(t)\right)<0 \\ \quad h_{\mathrm{IE}}\left(\boldsymbol{x}(t_{\mathrm{f}}), \boldsymbol{y}(t_{\mathrm{f}}), \boldsymbol{u}(t_{\mathrm{f}})\right)<0 \\ \quad \boldsymbol{u}_{\mathrm{L}} \le \boldsymbol{u}(t) \le \boldsymbol{u}_{\mathrm{U}} \\ \quad \boldsymbol{x}\left(t_0\right) = \boldsymbol{x}^0 \\ \quad \boldsymbol{x}\left(t_{\mathrm{f}}\right) = \boldsymbol{x}^{\mathrm{f}} \\ \quad t \in [t_0, t_{\mathrm{f}}] \end{cases} \tag{16}$$

where $\boldsymbol{x}$ is the state variable, $\boldsymbol{x} = [\psi_{\mathrm{d}}, P_{\mathrm{H}}, \psi_{\mathrm{z}}, V_{\mathrm{f}}]^T$; F is the dynamic model of CHP unit DAEs, corresponding to Eq. (1) to Eq. (8); g_{E} and g_{IE} are the equal and unequal path constraints, corresponding to Eqs. (9), (10) and (15); h_{E} and h_{IE} are the final value constraints at t_{f}, corresponding to Eq. (11); and the initial and termination boundary of the state variable $\boldsymbol{x}$ are calculated by Eqs. (13) and (14).

The dynamic optimization problem shown in Eq. (16) is discretized to generate a nonlinear planning problem, which can be solved directly by the SQP method [10]. However, in the application of the optimal control problem, selecting the Legendre-Gauss (LG) points less will cause the result of poor accuracy. Selecting LG point more will lead to the calculation of the amount of exponential growth. Although the direct shooting method can avoid the above drawbacks, its global search ability is poor. Selecting the initial value of the variable improperly often leads to the problem being trapped in local minimization.

To tackle these issues, hybrid serial optimization is employed by combining the direct shooting method and the Gauss pseudospectral method. The direct shooting method is used to compute the optimal exact solution. The Gauss pseudospectral method computes the strategy's initial solution. The hybrid optimization strategy combines the benefits of both methods. It has a faster convergence rate, a higher solution accuracy, and less reliance on the initial values of the variables.

3.2 Gauss Pseudospectral Method Calculates Feasible Initial Solutions

Time Domain Transformation. The time interval of the optimal control problem is $[t_0, t_\mathrm{f}]$, which is transformed $[-1,1]$ using the Gauss pseudospectral method. The formula is as follows:

$$\tau = \frac{2t}{t_\mathrm{f} - t_0} - \frac{t_\mathrm{f} + t_0}{t_\mathrm{f} - t_0} \tag{17}$$

where τ is the dimensionless time in $[-1,1]$.

Discretization of State and Control variables. In order to utilize a function to approximate the state variables, K LG collocation points ($\tau_i (i = 1, 2, \cdots, K)$) and $\tau_0 = -1$ are chosen as discrete nodes to form the $K+1$ Lagrange interpolating polynomial basis functions ($L_i(\tau)(i = 0, 1, \cdots, K)$).

$$\boldsymbol{x}(\tau) \approx \boldsymbol{X}(\tau) = \sum_{i=0}^{K} L_i(\tau) \boldsymbol{X}(\tau_i) \tag{18}$$

$$L_i(\tau) = \prod_{j=0, j \neq i}^{K} \frac{(\tau - \tau_j)}{(\tau_i - \tau_j)} \tag{19}$$

The Lagrange interpolating polynomial $\tilde{L}_i(\tau)(i = 1, 2, \cdots, K)$ are used as basis functions to approximate the control variables.

$$\boldsymbol{u}(\tau) \approx \boldsymbol{U}(\tau) = \sum_{i=1}^{K} \tilde{L}_i(\tau) \boldsymbol{U}(\tau_i) \tag{20}$$

$$\tilde{L}_i(\tau) = \prod_{j=1, j \neq i}^{K} \frac{(\tau - \tau_j)}{(\tau_i - \tau_j)} \tag{21}$$

Discretization of Terminal State Constraints. The terminal state is represented as an integral form of discrete state variables and control variables, and then Eq. (22) can be approximately obtained by the Gauss integral.

$$\boldsymbol{X}_\mathrm{f} = \boldsymbol{X}_0 + \frac{t_\mathrm{f} - t_0}{2} \sum_{k=1}^{K} \mu_k F(\boldsymbol{X}_k, \boldsymbol{U}_k, \tau_k; t_0, t_\mathrm{f}) \tag{22}$$

where $\mu_k = \int_{-1}^{1} L_k(\tau) d\tau$ is the Gaussian weight and τ_k is the LG collocation point.

Transformation of Dynamic Differential Equation. The derivative of Eq. (18) is first performed and then the time discretization points are brought into the equation to obtain the derivative formula of the state equation as shown in Eq. (23).

$$\dot{\boldsymbol{x}}(\tau_k) \approx \dot{\boldsymbol{X}}(\tau_k) = \sum_{i=0}^{K} \dot{L}_i(\tau_k)\boldsymbol{X}(\tau_i) = \sum_{i=0}^{K} D_{ki}\boldsymbol{X}(\tau_i) \tag{23}$$

where the differential matrix $\boldsymbol{D} \in \boldsymbol{R}^{K\times(K+1)}$ is defined by Eq. (24).

$$D_{ki} = \dot{L}_i(\tau_k) = \sum_{l=0,l\neq i}^{K} \frac{\prod_{j=0\, j\neq i,l}^{K} \tau_k - \tau_j}{\prod_{j=0\, j\neq i}^{K} \tau_i - \tau_j} \tag{24}$$

Eq. (23) is substituted into the discrete expression of the state constraint in Eq. (16) at the collocation points.

$$\sum_{i=0}^{K} D_{ki}\boldsymbol{X}_i - \frac{t_\mathrm{f} - t_0}{2} F(\boldsymbol{X}_k, \boldsymbol{U}_k, \tau_k; t_0, t_\mathrm{f}) = 0 \tag{25}$$

where $k = 1, 2, \cdots, K$, $i = 0, 1, \cdots, K$, $\boldsymbol{X}_k \equiv \boldsymbol{X}(\tau_k)$, $\boldsymbol{U}_k \equiv \boldsymbol{U}(\tau_k)$.

Objective Function. In the objective function, the output variable $\boldsymbol{y} = [\psi_\mathrm{t}, P_\mathrm{H}, m_\mathrm{H}]^T$ can be transformed from Eqs. (1)-(8) into an expression about the state variable $\boldsymbol{x} = [\psi_\mathrm{d}, P_\mathrm{H}, \psi_\mathrm{z}, V_\mathrm{f}]^T$. Using Gauss integration to approximate the integral term of the objective function Eq. (12) in the optimal control problem. The approximate objective function in discrete form is obtained as:

$$J = \int_{t_0}^{t_\mathrm{f}} F(\boldsymbol{x}, \boldsymbol{u})dt = \frac{t_\mathrm{f} - t_0}{2} \sum_{k=1}^{K} \mu_k F(\boldsymbol{X}_k, \boldsymbol{U}_k) \tag{26}$$

3.3 Direct Shooting Method Calculates Ideal Exact Answers

The direct shooting method is a parameter optimization method that solely uses discrete control variables [10]. It discretizes the time-continuous optimum control problem to transform the original problem into a nonlinear programming problem. Then discretizes the continuous time into K segments according to Eq. (27).

$$t_0 = t_1 < t_2 < \cdots < t_K = t_\mathrm{f} \tag{27}$$

The design variables are the relevant control variables on the discrete-time nodes from Eq. (28). The values of the control variables across nearby time nodes can be determined via cubic spline interpolation.

$$\boldsymbol{u}=\left(\boldsymbol{U}_1,\boldsymbol{U}_2,\cdots,\boldsymbol{U}_K\right) \tag{28}$$

After generating a set of design variables, the control variables are obtained through interpolation, and then the state equation is integrated to obtain the state variables, which can be used to solve the objective function and constraint equation. For solving optimization control problems, only the end time and control variables need to be used as design variables together. The discrete-time format is determined by the relative independent variables of the specific model equations, which can be actual time, dimensionless time, and energy parameters. A discrete-time process is a dynamic process in an optimization process.

Through the above process, the CHP unit variable load optimal control problem is discretized into a nonlinear programming problem in the form of algebraic constraints. The objective function (Eq. (26)) is minimized by solving the discrete control variables $\boldsymbol{u}$. At the same time, the state constraint equations (Eq. (25)), terminal state constraints (Eq. (22)), boundary condition constraints (Eqs. (13)-(14)), and inequality path constraints (Eq. (15)) are satisfied.

4 Simulation results analysis

4.1 CHP Unit Variable Load Simulation Scenario

In this article, the parameters of the extraction CHP unit under various working conditions in [5] are utilized for modeling and simulation analysis. The unit's control system parameters are provided in Table 1. Table 2 displays the case data of the unit's variable load demands that are common in integrated energy system applications. The unit startup takes 300s to adjust. The adjustment time of the variable load control process from the initial value to the terminal value is 300s.

Table 1. CHP unit control system parameters.

V_T control parameters	V_B control parameters	V_H control parameters	Other parameters
K_{PT} = -0.9	K_{PB} = 0.1	K_{PH} = 0.01	K = 0.5
K_{IT} = -1	K_{IB} = 0.01	K_{IH} = 10	T_c = 15

Table 2. Cases of variable load demand of CHP unit.

Case	Initial value (P_H^0,ψ_t^0,m_H^0)	Terminal value (P_H^f,ψ_t^f,m_H^f)
Case 1: Heating condition downward step	(250,16.67,400)	(220,16.67,400)
Case 2: Heating condition upward step	(230,16.67,380)	(260,16.67,390)
Case 3: Pure condensation condition downward step	(260,16.67,0)	(230,16.67,0)

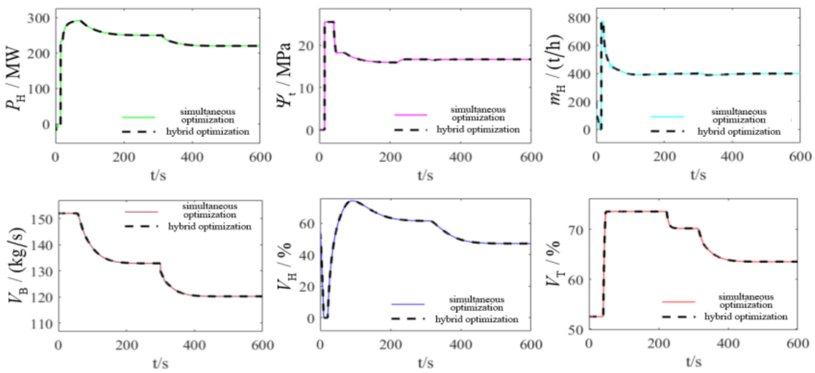
4.2 Simulation Results and Analysis

Validity Analysis of Simulation Results. The output curves and control trajectories of CHP unit from start-up to operation and then to variable load in different cases are shown in Fig. 1. Under heating condition (Case 1, Case 2), the simulation results of this paper are basically consistent with the output variable curve and control variable trajectory obtained from literature [11] (simultaneous optimization solution), indicating that the hybrid optimization strategy proposed in this paper is feasible. Under pure coagulation condition (Case 3), the results obtained by the two optimization algorithms are also the same. In three types of variable load cases, the CHP unit load can be stably and accurately adjusted to the required indicator value, and all have achieved the optimal control effect. The data shows that the proposed optimization strategy has good adaptability to different operating conditions and varying load demands throughout the entire simulation time domain.

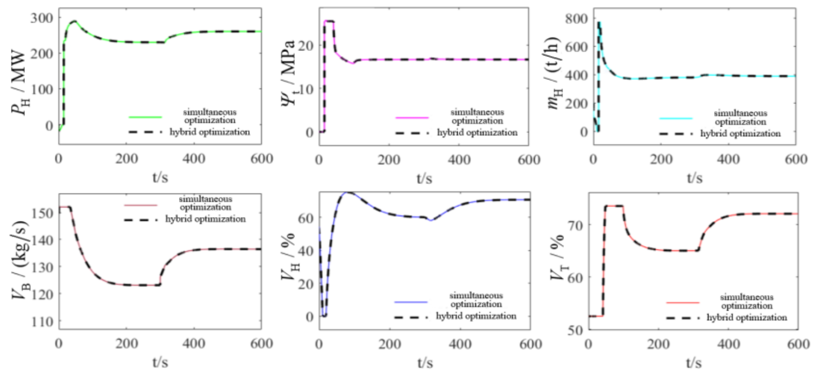
Analysis of Computational Performance Indicators. Under different operating conditions and variable load demands, the data in Table 3 shows that the optimal control terminal values of Gauss pseudospectral method and hybrid optimization strategy are almost the same. However, the number of iterations and time consumed for solving differ significantly. Hybrid optimization can reduce the difference between the initial value and the optimal solution, allowing it to swiftly converge to the optimal solution. Refer to Case 1, compared to solving by the Gauss pseudospectral method alone, the terminal value of the hybrid optimization strategy solution (219.9, 16.67, 399.6) is slightly lower than the demanded value (220, 16.67, 400). However, computation time is shortened by about 32% while the number of iterations is about 26%. The feasibility errors are within the calculation requirements.

Table 3. Computational performance of different methods.

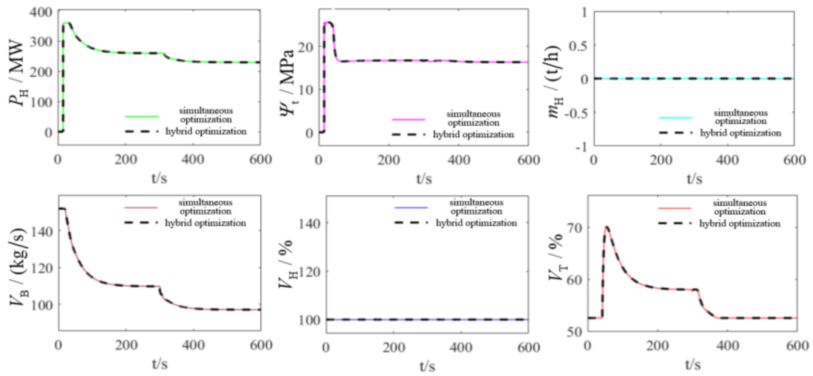
Case	Indicators	Gauss pseudospectral method	Hybrid optimization strategy
Case1	Solution time	3.05s	2.01s
	Iterations	59	40
	Feasibility error	5.34×10^{-12}	1.51×10^{-10}
	Terminal values	(220,16.67,400)	(219.9,16.67,399.6)
Case2	Solution time	3.31s	2.12s
	Iterations	65	45
	Feasibility error	5.52×10^{-12}	1.66×10^{-10}
	Terminal values	(260,16.67,390)	(260,16.67,390.1)
Case3	Solution time	2.88s	1.35s
	Iterations	54	32
	Feasibility error	5.81×10^{-12}	1.72×10^{-10}



(a) Case 1



(b) Case 2



(c) Case 3

Fig. 1. Results of dynamic optimization of unit variable load process under different conditions.

5 Conclusion

In this paper, the dynamic optimization problem of load control process for CHP units was investigated by combining the Gauss pseudospectral method and the direct shooting method. The simulation results indicate that the serial hybrid optimization method can rapidly and accurately plan out the change curves of output variables and the trajectories of control variables of CHP units, guiding the actual operational control of CHP units. Furthermore, it provides a solving approach for other optimization control problems.

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