Observer-Based Finite-Time Consensus Control for Multi-Agent Systems with Non-Affine Faults

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Abstract. In this paper, the consensus control problem of high order nonlinear multi-agent systems with non-affine faults, unmeasurable states, uncertain control coefficients and external disturbances is studied. Under the directed topology conditions, an observer-based finite-time control method based on adaptive backstepping is proposed. A state observer based on neural network is introduced to estimate the unmeasurable system state variables. In order to solve the "explosion of complexity" problem of backstepping method, a finite time command filter is introduced, and error compensation signals are designed to compensate the filter error. The Butterworth low-pass filter is used to avoid the algebraic ring problem in the control law. The finite-time stability criterion and Lyapunov stability theorem are utilized to analyze that all signals of the closed-loop system are bounded in finite time. Finally, the effectiveness of the presented control strategy is illustrated by a simulation example.

Keywords: Command filtered backstepping, Finite-time control, Multi-agent systems, Neural observer, Non-affine faults

1 Introduction

In the past few decades, as the foundation of cooperation and coordination among agents, the consensus problem has received extensive attention in the research of multi-agent systems (MASs) [1-3]. Consensus control within the leader-following framework is a typical research direction[4], which aims to achieve state consensus among agents under the condition that only part of agents can directly access the leader's signal or reference signal.

In the design of consensus control protocols for nonlinear MASs, adaptive backstepping is a commonly used method[5-6]. It systematizes structures the process of Lyapunov function and controller design through recursive design. However, the derivative of the previous step is required in each step of the backstepping method, which lead to an increase in computational complexity with the order of the system. By merging the command filter approach, the "explosion of complexity" problem can be solved.

In the consensus control problem, convergence rate is generally considered to be an important factor. But many study on consensus control in existing literatures are based on asymptotic stability[7-8], which ensures system stability only as time tends to infinity. In contrast, finite-time control protocols can provide faster convergence rate, higher tracking accuracy, and better performance[9-12].

In practical systems, the system state variables are usually partial measurable. In such cases, methods based on the full observability of agent states and state feedback cannot be applied. A feasible approach is to design a state observer to estimate the unknown part of the system state. For systems with nonlinear terms, a state observer based on neural network is effective[13-15]. Besides, the control coefficients are often uncertain in practical, which may affect the effect of the control protocol[12,16].

In addition, due to the large scale and high complexity of MASs, some faults are prone to occur during the operation of real systems, which may lead to a decline in control performance or even cause the system to fail directly. Therefore, to ensure the long-term stable operation of the system, it is crucial to consider the fault tolerance of the system in the design of control protocol[16-18]. Most of the above literatures consider linear fault models, which may not reflect the nonlinear fault characteristics of complex systems well, while non-affine fault models can present a wider range of fault conditions in practical applications more accurately[15,19-20].

Motivated by the above observations, a finite-time fault tolerance control scheme based on neural observer is presented for nonlinear MASs with non-affine faults and uncertain control coefficients in this paper. The main contributions of this paper are listed as follows.

1. An advanced observer-based finite-time adaptive tracking control algorithm, combining adaptive backstepping control and radial basis function neural network (RBFNN) , is proposed for uncertain nonlinear MASs subject to non-affine faults, uncertain control coefficients and external disturbances under directed graphs.
2. In contrast to traditional linear fault model, the non-affine fault model in this paper can reflect faults in practical systems more accurately. Besides, the uncertain control coefficients considered in this paper is more practical in real systems than many other existing literatures, whose control coefficients are regarded as known constant.
3. To address the "explosion of complexity" problem raised by backstepping method, the command filter mechanism is introduced in this paper, and compensation signals are established to compensate the filter error. In addition, a Butterworth low-pass filter (BLPF) is utilized to solve the algebraic ring problem in the control law.

The remainder of this paper is organized as follows. In Section 2, the communication graph, the considered system and faults model, some useful assumptions and lemmas are introduced. Then, a neural observer and a corresponding observer-based finite-time adaptive fault-tolerant control protocol is derived in Section 3. In Section 4, the stability analysis is established. In Section 5, a simulation example is provided to illustrate the effectiveness of our proposed scheme. Finally, a conclusion is drawn in Section 6.

2 Preliminaries and problem description

2.1 Graph theory

The information interaction topology between agents is described by a fixed directed graph $G \triangleq (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} \triangleq \{1, \ldots, N\}$ is the vertex set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ indicates the edge set. The edge $(i, j) \in \mathcal{E}$ denotes that agent *j* can obtain information from agent *i*, but not necessarily vice versa. For the edge (i, j), *i* is the parent node and *j* is the child node. The neighbors collection of agent *j* is represented as $\mathcal{N}_j = \{i \mid (i, j) \in \mathcal{E}\}$. $A = [a_{i,j}] \in R^{N \times N}$ represents the adjacency matrix, in which $a_{i,j}$ is a positive weight if and only if $(j, i) \in \mathcal{E}$, otherwise $a_{i,j} = 0$. The in-degree matrix is represented as $\mathcal{D}$, which is a diagonal matrix with diagonal elements $d_i = \Sigma_{j \in \mathcal{N}_i} a_{i,j}$. The Laplacian matrix is expressed as $L = \mathcal{D} - A$. The leader adjacency matrix is $B = diag\{b_1, \ldots, b_n\}$, in which $b_i = 1$ if and only if the node *i* can receive the leader's information, otherwise $b_i = 0$. If the directed graph *G* contains a directed spanning tree, it indicates that there is at least one node with a directed path to all other nodes.

2.2 Problem description

In this paper, the MAS consisting of one leader and *N* followers is considered. The *i*-th follower is modeled as

$$\begin{aligned} \dot{x}_{i,\ell} &= x_{i,\ell+1} \\ \dot{x}_{i,n} &= \rho_i(t) u_i + f_i(x_i) + \beta_i(t - T_0) h_i(x_i, u_i) + \Delta_i(t) \\ y_i &= x_{i,1} \end{aligned} \tag{1}$$

where $\ell = 1, 2, \ldots, n-1$. $x_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,n}]^{\mathrm{T}}$ and y_i are the system state vector and output of the *i*-th follower, respectively. And only the output is measurable while other states variables are not available. $f_i(x_i)$ is an unknown nonlinear function. $\Delta_i(t)$ denotes the uncertain external disturbances. u_i is the control signal with an uncertain nonlinear control coefficient $\rho_i(t)$ satisfied $0 < \rho_i(t) \le \overline{\rho}_i$, where $\overline{\rho}_i$ is a positive constant. $h_i(x_i, u_i)$ stands for fault-induced unknown change and $\beta_i(t - T_0)$ represents the time profile with the form

$$\beta_i(t - T_0) = \begin{cases} 0 & t < T_0 \\ 1 - e^{-r_{a,i}(t - T_0)} & t \ge T_0 \end{cases}$$

where $r_{a,i} > 0$ and T_0 indicate the fault evolution rate and unknown fault occurrence time, respectively.

The control objective is to design a finite-time fault-tolerant control (FTC) scheme to guarantee the desired tracking performance of system (1), and all signals in the closed-loop system are semi-globally uniformly ultimately bounded (SGUUB) in finite time. To achieve such objective, some necessary preliminaries are listed as follow.

Assumption 1.[16] The directed graph G contains a spanning tree and the root node n_r can directly access the full information of reference signal $y_r(t)$, that is $b_r = 1$.
Assumption 2.[16] The disturbance satisfies $|\Delta_i(t)| \le \overline{\Delta}_i$, where $\overline{\Delta}_i$ is an unknown positive constant.
Assumption 3.[14] The partial derivative $[(\partial f_i)/(\partial x_{i,\ell})]$ satisfies the inequality:

$$\underline{f}_{i,\ell} \le (\partial f_i)/(\partial x_{i,\ell}) \le \overline{f}_{i,\ell}$$

where $\underline{f}_{i,\ell}$ and $\overline{f}_{i,\ell}$ are known constants.
Assumption 4.[15] The reference signal y_r and its first-order derivative $\dot{y}_r$ are continuous and bounded.
Assumption 5.[15] For $\beta_i(t-T_0)h_i(x_i,u_i)$, the following inequality is satisfied

$$|\beta_i(t-T_0)h_i(x_i,u_i)| \le |\overline{g}_i(x_i,u_i)|$$

where $\overline{g}_i(x_i,u_i)$ is an unknown function.
Lemma 1.[9] An extended Lyapunov description of finite-time stability can be given with the form of $\dot{V}(x) + \lambda_1 V(x) + \lambda_2 V^{\gamma}(x) \le 0$, and the settling time can be given by

$$T_r \le t_0 + \frac{1}{\lambda_1(1-\gamma)} \ln \frac{\lambda_1 V^{1-\gamma}(t_0) + \lambda_2}{\lambda_2}$$

where λ_1, λ_2, $0 < \gamma < 1$ are positive constants.
Lemma 2.[10] Consider the system $\dot{x} = f(x)$, if there exist continuous function $V(x)$ and positive constants λ_1, λ_2, $0 < \gamma < 1$, $0 < \eta < \infty$ such that $\dot{V}(x) \le -\lambda_1 V(x) - \lambda_2 V^{\gamma}(x) + \eta$, then the trajectory of system is practical finite-time stable, and

$$\left\{\lim_{t\to T_r} |V(x)| \le \min\left\{\eta/(1-\theta_0)\lambda_1, \left(\eta/(1-\theta_0)\lambda_2\right)^{1/\gamma}\right\}\right\}$$

where $0 < \theta_0 < 1$ is a constant. The settling time is bounded as

$$T_r \le \max\left\{t_0 + \frac{1}{\theta_0\lambda_1(1-\gamma)} \ln \frac{\theta_0\lambda_1 V^{1-\gamma}(t_0) + \lambda_2}{\lambda_2}, t_0 + \frac{1}{\lambda_1(1-\gamma)} \ln \frac{\lambda_1 V^{1-\gamma}(t_0) + \theta_0\lambda_2}{\theta_0\lambda_2}\right\}$$

Lemma 3.[15] Let $S(\overline{z}_q) = [s_1(\overline{z}_q), \ldots, s_n(\overline{z}_q)]^{\mathrm{T}}$ be a basis function vector of RBFNN with $\overline{z}_q = [z_1, \ldots, z_q]^{\mathrm{T}}$. For any integer $0 < p \le q$, $S^{\mathrm{T}}(\overline{z}_q)S(\overline{z}_q) \le S^{\mathrm{T}}(\overline{z}_p)S(\overline{z}_p)$.
Lemma 4.[15] There exist $w > 0$ and $z \in R$, such that

$$|z| \le w + \frac{z^2}{\sqrt{z^2 + w^2}}$$

3 Design of observer and controller

For simplicity, we define some necessary symbols and parameters as follows: $(\hat{\bullet})$ is the estimation of $(\bullet)^*$ and $(\tilde{\bullet}) = (\bullet)^* - (\hat{\bullet})$. $\|\bullet\|$ represents the Euclidean norm. $\gamma_{i,1}$, $\gamma_{i,2}$, $\tau_{i,1}$, $\tau_{i,2}$, $\tau_{i,3}$, $\kappa_{i,\ell}$, $c_{i,\ell}$, $w_{i,\ell}$, $h_{i,\ell}$, $k_{i,\ell}$, p, $\lambda_{i,1}$, $q_{i,0}$, $q_{i,\ell}$, r_i, $\mu_{1,i}$, $\mu_{2,i}$, $\sigma_{1,i}$, $\sigma_{2,i}$, $\sigma_{3,i}$, $\sigma_{4,i}$ are positive parameters for $i = 1, \ldots, N$ and $\ell = 1, \ldots, n$.

3.1 Neural observer design

For unmeasurable states in system (1), a neural observer is constructed as follows

$$\begin{aligned}\dot{\hat{x}}_{i,\ell} &= \hat{x}_{i,\ell+1} + l_{i,\ell}\left(y_i - \hat{x}_{i,1}\right)\\ \dot{\hat{x}}_{i,n} &= l_{i,n}\left(y_i - \hat{x}_{i,1}\right) + u_i + \hat{\theta}_i^{\mathrm{T}} S_i\left(\hat{X}_i\right) + \hat{\upsilon}_{1,i}\psi_{1,i}\left(\hat{X}_i, u_i^f\right) + \upsilon_{2,i}\psi_{2,i}\left(u_i^f\right)\end{aligned} \tag{2}$$

where $\ell = 1,\ldots,n-1$, $\hat{X}_i = \left[x_{i,1}, \hat{x}_{i,2}, \ldots, \hat{x}_{i,n}\right]^{\mathrm{T}}$, $\hat{x}_{i,\ell}$ is the estimation of $x_{i,\ell}$. $l_{i,\ell}$ denotes the observer gain. $\hat{\theta}_i$, $\hat{\upsilon}_{1,i}$, $\hat{\upsilon}_{2,i}$ are adaptive parameter vector utilized to estimate the unknown ideal RBFNN weight vector θ_i^*, $\upsilon_{1,i}^*$, $\upsilon_{2,i}^*$, respectively. S_i, $\psi_{1,i}$, $\psi_{2,i}$ denotes basis function vectors. $u_i^f \approx u_i$ is the filtered signal generated by a Butterworth low-pass filter (BLPF) $u_i^f = H_i^L(s)u_i$, whose parameters are shown in [21].

Design the estimation error vector $e_i = \left[x_{i,1} - \hat{x}_{i,1}, \ldots, x_{i,n} - \hat{x}_{i,n}\right]^{\mathrm{T}} = \left[e_{i,1}, \ldots, e_{i,n}\right]^{\mathrm{T}}$. From (1) and (2), the estimation error dynamics is represented as

$$\begin{aligned}\dot{e}_i = A_i e_i + R_i\Big(& f_i(x_i) - f_i(\hat{X}_i) + f_i(\hat{X}_i) - \hat{\theta}_i^{\mathrm{T}} S_i(\hat{X}_i) + \beta_i(t - T_0)h_i(x_i, u_i)\\ &-\hat{\upsilon}_{1,i}\psi_{1,i}(\hat{X}_i, u_i^f) - \hat{\upsilon}_{2,i}\psi_{2,i}(u_i^f) + \Delta_i(t) + (\rho_i(t) - 1)u_i\Big)\end{aligned} \tag{3}$$

where $A_i = A_{0,i} - L_i C_i$, $A_{0,i} = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}$, $L_i = [l_{i,1}, \ldots, l_{i,n}]^{\mathrm{T}}$, $C_i = [1, 0, \ldots, 0]$, $R_i = [0, \ldots, 0, 1]^{\mathrm{T}}$, and I_{n-1} is the $n-1$ order identity matrix.

The unknown nonlinear function $f_i(\hat{X}_i)$ is estimated by RBFNN as follows

$$f_i\left(\hat{X}_i\right) = \theta_i^{*\mathrm{T}} S_i\left(\hat{X}_i\right) + \delta_i, \quad |\delta_i| \le \bar{\delta}_i \tag{4}$$

By differential mid-value theorem, one can obtain

$$f_i\left(x_i\right) - f_i\left(\hat{X}_i\right) = F_i e_i \tag{5}$$

where $F_i = [0, F_{i,2}, \ldots, F_{i,n}]$, $F_{i,\ell} = [(\partial f_i)/(\partial \varsigma_{i,\ell})]$, $\varsigma_{i,\ell} \in [x_{i,\ell}, \hat{x}_{i,\ell}]$, $2 \le \ell \le n$. Furthermore, define $J_i = R_i F_i$ for the convenience of subsequent steps.

For the stability analysis of estimation error dynamics, construct a Lyapunov function candidate as $V_{e,i} = e_i^{\mathrm{T}} P_i e_i$ with P_i being a symmetric positive definite matrix to be decided. By (3)-(5), the time derivative of $V_{e,i}$ can be represented as

$$\begin{aligned}\dot{V}_{e,i} = e_i^{\mathrm{T}}\left(A_i^{\mathrm{T}} P_i + P_i A_i\right)e_i + 2e_i^{\mathrm{T}} P_i R_i\Big(& F_i e_i + \tilde{\theta}_i^{\mathrm{T}} S_i\left(\hat{X}_i\right) + \delta_i + \beta_i\left(t - T_0\right)h_i\left(x_i, u_i\right)\\ &-\hat{\upsilon}_{1,i}\psi_{1,i}\left(\hat{X}_i, u_i^f\right) - \hat{\upsilon}_{2,i}\psi_{2,i}\left(u_i^f\right) + \Delta_i\left(t\right) + \left(\rho_i\left(t\right) - 1\right)u_i\Big)\end{aligned} \tag{6}$$

Applying Assumption 2, Assumption 5 and Young's inequality, and utilizing RBFNN to estimate unknown terms, one can obtain

$$\Delta_i\left(t\right) \le \frac{1}{2\gamma_{i,1}^2}\bar{\Delta}_i^2 + \frac{1}{2}\gamma_{i,1}^2 \tag{7}$$

$$\beta_i\left(t - T_0\right)h_i\left(x_i, u_i\right) \le \frac{1}{2\gamma_{i,2}^2}\bar{g}_i^2\left(x_i, u_i\right) + \frac{1}{2}\gamma_{i,2}^2 \tag{8}$$

$$\frac{1}{2\gamma_{i,1}^2}\bar{\Delta}_i^2 + \frac{1}{2\gamma_{i,2}^2}\bar{g}_i^2\left(x_i, u_i\right) = \upsilon_{1,i}^{*\mathrm{T}}\psi_{1,i}\left(\hat{X}_i, u_i^f\right) + \varepsilon_{1,i}, \quad \left|\varepsilon_{1,i}\right| \le \bar{\varepsilon}_{1,i} \tag{9}$$

$$\left(\bar{\rho}_i - 1\right)\|u_i\| + \frac{1}{2}\left(\gamma_{i,1}^2 + \gamma_{i,2}^2\right) = \upsilon_{2,i}^{*\mathrm{T}}\psi_{2,i}\left(u_i^f\right) + \varepsilon_{2,i}, \ \left|\varepsilon_{2,i}\right| \le \bar{\varepsilon}_{2,i} \tag{10}$$

From the property of RBFNN that $S_i^{\mathrm{T}} S_i \le d_{S,i}$ with $d_{S,i}$ being the dimension number of S_i, one can further obtain that

$$2e_i^{\mathrm{T}} P_i R_i \left(\tilde{\theta}_i^{\mathrm{T}} S_i\left(\hat{X}_i\right) + \delta_i\right) \le 2\tau_{i,1} e_i^{\mathrm{T}} P_i e_i + \frac{\|P_i\|}{\tau_{i,1}}\left(d_{S,i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \bar{\delta}_i^2\right) \tag{11}$$

$$2e_i^{\mathrm{T}} P_i R_i \left(\tilde{\upsilon}_{k,i}^{\mathrm{T}}\psi_{k,i} + \varepsilon_{k,i}\right) \le 2\tau_{i,k} e_i^{\mathrm{T}} P_i e_i + \frac{\|P_i\|}{\tau_{i,k}}\left(d_{\psi k,i}\tilde{\upsilon}_{k,i}^{\mathrm{T}}\tilde{\upsilon}_{k,i} + \bar{\varepsilon}_{k,i}^2\right), k = 1,2 \tag{12}$$

where $d_{S,i}$, $d_{\psi 1,i}$, $d_{\psi 2,i}$ indicate the dimension of S_i, $\psi_{1,i}$ and $\psi_{2,i}$, respectively.

Substitute (7)-(12) into (6) yields

$$\begin{aligned}\dot{V}_{e,i} \le\ & e_i^{\mathrm{T}}\left(A_i^{\mathrm{T}} P_i + P_i A_i + 2\tau_i P_i + P_i J_i + J_i^{\mathrm{T}} P_i\right)e_i + \frac{\|P_i\|}{\tau_{i,1}}\left(d_{S,i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i + \bar{\delta}_i^2\right) \\ & + \frac{\|P_i\|}{\tau_{i,2}}\left(d_{\psi 1,i}\tilde{\upsilon}_{1,i}^{\mathrm{T}}\tilde{\upsilon}_{1,i} + \bar{\varepsilon}_{1,i}^2\right) + \frac{\|P_i\|}{\tau_{i,3}}\left(d_{\psi 2,i}\tilde{\upsilon}_{2,i}^{\mathrm{T}}\tilde{\upsilon}_{2,i} + \bar{\varepsilon}_{2,i}^2\right)\end{aligned} \tag{13}$$

where $\tau_i = \tau_{i,1} + \tau_{i,2} + \tau_{i,3}$. From (13), the following matrix inequality needs to be satisfied for the stability of the error dynamics

$$A_i^{\mathrm{T}} P_i + P_i A_i + 2\tau_i P_i + P_i J_i + J_i^{\mathrm{T}} P < 0 \tag{14}$$

From Assumption 3, there exists functions $\chi_{i,\ell}(t), (0 \le \chi_{i,\ell} \le 1)$ such that

$$F_{i,\ell} = \chi_{i,\ell}\underline{f}_{i,\ell} + \left(1 - \chi_{i,\ell}\right)\bar{f}_{i,\ell} \tag{15}$$

As a result, the element of matrix J_i can be rewritten to the above form. Thus, by the convex combination theory, (14) can be converted to

$$A_i^{\mathrm{T}} P_i + P_i A_i + 2\tau_i P_i + P_i B_i + B_i^{\mathrm{T}} P < 0 \tag{16}$$

where $B_i \in \Omega_i$, $\Omega_i = \left\{B_i \left| [B_i]_{n,\ell} = \underline{f}_{i,\ell} \ or\ \bar{f}_{i,\ell}; 2 \le \ell \le n\right.\right\}$

Remark 1: By introducing $N_i = P_i L_i$ and above steps, (14) can be simplified into linear matrix inequality (LMI) form. Thus, L_i can be obtained by solving the LMIs, and the solvability of LMIs guarantees the stability of the error dynamics[14].

3.2 Finite-time controller design

First, the coordinate transformation of the *i*-th follower is described below

$$\begin{aligned} v_{i,1} &= \sum_{j \in N_i} a_{i,j}\left(y_i - y_j\right) + b_i\left(y_i - y_r\right) \\ v_{i,\ell} &= \hat{x}_{i,\ell} - \bar{\alpha}_{i,\ell}, \ \ell = 2, \ldots, n \end{aligned} \tag{17}$$

where $v_{i,1}$, $v_{i,\ell}$ represents the synchronization error and the error surface, respectively. y_r denotes the reference signal from the leader. N_i indicates the set of neighbor nodes of *i*. $a_{i,j}$ is the communication topology parameter between node *i* and node *j*. b_i indicates whether the reference signal is accessible for node *i*. $\bar{\alpha}_{i,\ell}$ is the output signal of a finite-time command filter inspired by [22], which if defined as

$$\begin{aligned} \omega_{i,\ell}\dot{\bar{\alpha}}_{i,\ell} &= \left|\alpha_{i,\ell-1} - \bar{\alpha}_{i,\ell}\right|^p \operatorname{sgn}\left(\alpha_{i,\ell-1} - \bar{\alpha}_{i,\ell}\right) + \alpha_{i,\ell-1} - \bar{\alpha}_{i,\ell} \\ \bar{\alpha}_{i,\ell}\left(0\right) &= \alpha_{i,\ell-1}\left(0\right) \end{aligned} \tag{18}$$

where $\omega_{i,\ell}$ and $0 < p < 1$ are positive constants. The virtual control law $\alpha_{i,\ell-1}$ is the input of the command filter, and the output is $\bar{\alpha}_{i,\ell}$.

The command filter will generate error $\bar{\alpha}_{i,\ell} - \alpha_{i,\ell-1}$, which will be eliminated by an error compensation mechanism. The compensated tracking error is defined as

$$z_{i,\ell} = v_{i,\ell} - \xi_{i,\ell},\ \ell = 1,\ldots,n \tag{19}$$

where $\xi_{i,\ell}$ is the error compensation signal designed as follow and $\xi_{i,\ell}(0) = 0$.

$$\begin{aligned}
\dot{\xi}_{i,1} &= \left(d_i + b_i\right)\left(\xi_{i,2} + \bar{\alpha}_{i,2} - \alpha_{i,1}\right) - c_{i,1}\xi_{i,1} - \frac{1}{2}\xi_{i,1} - \kappa_{i,1}\operatorname{sgn}\left(\xi_{i,1}\right) \\
\dot{\xi}_{i,2} &= \xi_{i,3} + \bar{\alpha}_{i,3} - \alpha_{i,2} - c_{i,2}\xi_{i,2} - \frac{1}{2}\xi_{i,2} - \left(d_i + b_i\right)\xi_{i,1} - \kappa_{i,2}\operatorname{sgn}\left(\xi_{i,2}\right) \\
\dot{\xi}_{i,\ell} &= \xi_{i,\ell+1} + \bar{\alpha}_{i,\ell+1} - \alpha_{i,\ell} - c_{i,\ell}\xi_{i,\ell} - \frac{1}{2}\xi_{i,\ell} - \xi_{i,\ell-1} - \kappa_{i,\ell}\operatorname{sgn}\left(\xi_{i,\ell}\right), \ell = 3,\ldots,n-1 \\
\dot{\xi}_{i,\ell} &= -c_{i,n}\xi_{i,n} - \frac{3}{2}\xi_{i,n} - \xi_{i,n-1} - \kappa_{i,n}\operatorname{sgn}\left(\xi_{i,n}\right)
\end{aligned} \tag{20}$$

where d_i is the in-degree of the *i*-th follower.

The virtual control signals in the following steps are constructed as

$$\alpha_{i,\ell} = -\frac{z_{i,\ell}\alpha_{i,\ell}^{*2}}{\sqrt{z_{i,\ell}^2\alpha_{i,\ell}^{*2} + w_{i,\ell}^2}},\ \ell = 1,\ldots,n \tag{21}$$

where

$$\begin{aligned}
\alpha_{i,1}^{*} &= \frac{1}{d_i + b_i}\left(c_{i,1}v_{i,1} + \frac{1}{2}v_{i,1} - \sum_{j\in N_i} a_{i,j}\hat{x}_{j,2} - b_i\dot{y}_r + h_{i,1}z_{i,1}^{2p-1}\right) + \frac{z_{i,1}\hat{\phi}_i W_{i,1}^{\mathrm{T}}\left(\bar{Z}_{i,1}\right)W_{i,1}\left(\bar{Z}_{i,1}\right)}{2\left(d_i + b_i\right)k_{i,1}^2} \\
\alpha_{i,2}^{*} &= c_{i,2}v_{i,2} + \frac{1}{2}v_{i,2} + \left(d_i + b_i\right)v_{i,1} - \dot{\bar{\alpha}}_{i,2} + h_{i,2}z_{i,2}^{2p-1} + \frac{z_{i,2}}{2k_{i,2}^2}\hat{\phi}_i W_{i,2}^{\mathrm{T}}\left(\bar{Z}_{i,2}\right)W_{i,2}\left(\bar{Z}_{i,2}\right) \\
\alpha_{i,\ell}^{*} &= c_{i,\ell}v_{i,\ell} + \frac{1}{2}v_{i,\ell} + v_{i,\ell-1} - \dot{\bar{\alpha}}_{i,\ell} + h_{i,\ell}z_{i,\ell}^{2p-1} + \frac{z_{i,\ell}}{2k_{i,\ell}^2}\hat{\phi}_i W_{i,\ell}^{\mathrm{T}}\left(\bar{Z}_{i,\ell}\right)W_{i,\ell}\left(\bar{Z}_{i,\ell}\right) \\
\alpha_{i,n}^{*} &= c_{i,n}v_{i,n} + \frac{3}{2}v_{i,n} + v_{i,n-1} - \dot{\bar{\alpha}}_{i,n} + h_{i,n}z_{i,n}^{2p-1} + \frac{z_{i,n}}{2k_{i,n}^2}\hat{\phi}_i W_{i,n}^{\mathrm{T}}\left(\bar{Z}_{i,n}\right)W_{i,\ell}\left(\bar{Z}_{i,n}\right)
\end{aligned} \tag{22}$$

and $0 < p < 1$. $\hat{\phi}_i$ and $W_{i,\ell}$ are RBFNN parameters defined in the following steps.

Step 1: Construct the Lyapunov function candidate as

$$V_{i,1} = \frac{1}{2}z_{i,1}^2 + \frac{1}{2\lambda_{i,1}}\tilde{\phi}_i^2 \tag{23}$$

The time derivative of $V_{i,1}$ is

$$\begin{aligned}
\dot{V}_{i,1} = z_{i,1}\Big(&\left(d_i + b_i\right)\left(z_{i,2} + e_{i,2} + \bar{\alpha}_{i,2} + \xi_{i,2} + \alpha_{i,1} - \alpha_{i,1}\right) \\
&-\sum_{j\in N_i} a_{i,j}\left(\hat{x}_{j,2} + e_{j,2}\right) - b_i\dot{y}_r - \dot{\xi}_{i,1}\Big) - \frac{1}{\lambda_{i,1}}\tilde{\phi}_i\dot{\hat{\phi}}_i
\end{aligned} \tag{24}$$

By Young's inequality, one can obtain

$$z_{i,1}\left(d_i + b_i\right)e_{i,2} \le \frac{\left(d_i + b_i\right)^2}{2q_{i,1}}z_{i,1}^2 + \frac{q_{i,1}}{2}e_i^{\mathrm{T}}e_i \tag{25}$$

$$-z_{i,1}\sum_{j\in N_i} a_{i,j}e_{j,2} \le \frac{z_{i,1}^2}{2q_{i,0}}\sum_{j\in N_i} a_{i,j}^2 + \frac{q_{i,0}}{2}\sum_{j\in N_i} e_j^{\mathrm{T}}e_j \tag{26}$$

Using RBFNN to approximate the nonlinear terms above yields

$$g_{i,1}=\frac{\left(d_i+b_i\right)^2}{2q_{i,1}}z_{i,1}^2+\frac{z_{i,1}^2}{2q_{i,0}}\sum_{j\in N_i}a_{i,j}^2=\varphi_{i,1}^{*\mathrm{T}}W_{i,1}\left(Z_{i,1}\right)+\Delta_{i,1},\ \left|\Delta_{i,1}\right|\le\overline{\Delta}_{i,1}\tag{27}$$

where $Z_{i,1}=[x_{i,1},x_{j,1},y_r,\xi_{i,1}]^{\mathrm{T}}$. From Lemma 3 we can further obtain

$$z_{i,1}g_{i,1}\le\frac{1}{2k_{i,1}^2}z_{i,1}^2\phi_i^*W_{i,1}^{\mathrm{T}}\left(\overline{Z}_{i,1}\right)W_{i,1}\left(\overline{Z}_{i,1}\right)+\frac{1}{2}k_{i,1}^2+\frac{1}{2}z_{i,1}^2+\frac{1}{2}\overline{\Delta}_{i,1}^2\tag{28}$$

where $\phi_i^*=\max\left\{\|\varphi_{i,1}^*\|^2,\ldots,\|\varphi_{i,n}^*\|^2\right\}$, $\overline{Z}_{i,1}=[x_{i,1},x_{j,1},y_r]^{\mathrm{T}}$.

From Lemma 4 and (21), substitute $\dot{\xi}_{i,1}$, $\alpha_{i,1}^*$ and (25)-(28) into (24) yields

$$\begin{aligned}\dot{V}_{i,1}\le&-c_{i,1}z_{i,1}^2+\kappa_{i,1}z_{i,1}\operatorname{sgn}\left(\xi_{i,1}\right)+\left(d_i+b_i\right)z_{i,1}z_{i,2}-h_{i,1}z_{i,1}^{2p}+\frac{q_{i,1}}{2}e_i^{\mathrm{T}}e_i\\&+\frac{q_{i,0}}{2}\sum_{j\in N_i}e_j^{\mathrm{T}}e_j+\frac{\tilde{\phi}_i}{\lambda_{i,1}}\left(\frac{\lambda_{i,1}z_{i,1}^2}{2k_{i,1}^2}W_{i,1}^{\mathrm{T}}\left(\overline{Z}_{i,1}\right)W_{i,1}\left(\overline{Z}_{i,1}\right)-\dot{\hat{\phi}}\right)+D_{i,1}\end{aligned}\tag{29}$$

where $D_{i,1}=\left(d_i+b_i\right)w_{i,1}+k_{i,1}^2/2+\overline{\Delta}_{i,1}^2/2$

Step $\ell\ \left(2\le\ell\le n-1\right)$***:*** Construct the Lyapunov function candidate as

$$V_{i,\ell}=V_{i,\ell-1}+\frac{1}{2}z_{i,\ell}^2\tag{30}$$

and the derivative of (30) is as follow

$$\dot{V}_{i,\ell}=\dot{V}_{i,\ell-1}+z_{i,\ell}\left(z_{1,\ell+1}+\overline{\alpha}_{i,\ell+1}+\xi_{i,\ell+1}-\alpha_{i,\ell}+\alpha_{i,\ell}+l_{i,\ell}e_{i,1}-\dot{\overline{\alpha}}_{i,\ell}-\dot{\xi}_{i,\ell}\right)\tag{31}$$

Similarly, the following derivation can be obtained

$$z_{i,\ell}l_{i,\ell}e_{i,1}\le\left(0.5l_{i,\ell}^2/q_{i,\ell}\right)z_{i,\ell}^2+0.5q_{i,\ell}e_i^{\mathrm{T}}e_i\tag{32}$$

$$g_{i,\ell}=\left(0.5l_{i,\ell}^2/q_{i,\ell}\right)z_{i,\ell}=\varphi_{i,\ell}^{*\mathrm{T}}W_{i,\ell}\left(Z_{i,\ell}\right)+\Delta_{i,\ell},\ \left|\Delta_{i,\ell}\right|\le\overline{\Delta}_{i,\ell}\tag{33}$$

$$z_{i,\ell}g_{i,\ell}\le\frac{z_{i,\ell}^2}{2k_{i,\ell}^2}\phi_i^*W_{i,\ell}^{\mathrm{T}}\left(\overline{Z}_{i,\ell}\right)W_{i,\ell}\left(\overline{Z}_{i,\ell}\right)+\frac{1}{2}k_{i,\ell}^2+\frac{1}{2}z_{i,\ell}^2+\frac{1}{2}\overline{\Delta}_{i,\ell}^2\tag{34}$$

where $Z_{i,\ell}=[\hat{X}_i,\xi_{i,\ell}]^{\mathrm{T}}$ and $\overline{Z}_{i,\ell}=[\hat{X}_i]^{\mathrm{T}}$.

Substituting $\dot{\xi}_{i,\ell}$, $\alpha_{i,\ell}^*$ and (32)-(34) into (31) yields

$$\begin{aligned}\dot{V}_{i,\ell}\le&\sum_{k=1}^{\ell}\left(-c_{i,k}z_{i,k}^2+\kappa_{i,k}z_{i,k}\operatorname{sgn}\left(\xi_{i,k}\right)+\frac{q_{i,k}}{2}e_i^{\mathrm{T}}e_i-h_{i,k}z_{i,k}^{2p}+D_{i,k}\right)\\&+\frac{\tilde{\phi}_i}{\lambda_{i,1}}\left(\sum_{k=1}^{\ell}\frac{\lambda_{i,1}z_{i,k}^2}{2k_{i,k}^2}W_{i,k}^{\mathrm{T}}\left(\overline{Z}_{i,k}\right)W_{i,k}\left(\overline{Z}_{i,k}\right)-\dot{\hat{\phi}}\right)+\frac{q_{i,0}}{2}\sum_{j\in N_i}e_j^{\mathrm{T}}e_j+z_{i,\ell}z_{i,\ell+1}\end{aligned}\tag{35}$$

where $D_{i,\ell}=k_{i,\ell}^2/2+\overline{\Delta}_{i,\ell}^2/2+w_{i,\ell}$.

Step n: Consider the Lyapunov function candidate as

$$V_{i,n}=V_{i,n-1}+\frac{1}{2}z_{i,n}^2+\frac{1}{2r_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i+\frac{1}{2\mu_{1,i}}\tilde{\upsilon}_{1,i}^{\mathrm{T}}\tilde{\upsilon}_{1,i}+\frac{1}{2\mu_{2,i}}\tilde{\upsilon}_{2,i}^{\mathrm{T}}\tilde{\upsilon}_{2,i}\tag{36}$$

Differentiating $V_{i,n}$ yields

$$\begin{aligned}\dot{V}_{i,n}=&\dot{V}_{i,n-1}+z_{i,n}\Big(l_{i,n}e_{i,1}+u_i+\theta_i^{*\mathrm{T}}S_i\left(\hat{X}_i\right)-\tilde{\theta}_i^{\mathrm{T}}S_i\left(\hat{X}_i\right)+\upsilon_{1,i}^{*\mathrm{T}}\psi_{1,i}\left(\hat{X}_i,u_i^f\right)-\tilde{\upsilon}_{1,i}^{\mathrm{T}}\psi_{1,i}\left(\hat{X}_i,u_i^f\right)\\&+\upsilon_{2,i}^{*\mathrm{T}}\psi_{2,i}\left(u_i^f\right)-\tilde{\upsilon}_{2,i}^{\mathrm{T}}\psi_{2,i}\left(u_i^f\right)-\dot{\overline{\alpha}}_{i,n}-\dot{\xi}_{i,n}\Big)-\frac{1}{r_i}\tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i-\frac{1}{\mu_{1,i}}\tilde{\upsilon}_{1,i}^{\mathrm{T}}\dot{\hat{\upsilon}}_{1,i}-\frac{1}{\mu_{2,i}}\tilde{\upsilon}_{2,i}^{\mathrm{T}}\dot{\hat{\upsilon}}_{2,i}\end{aligned}\tag{37}$$

Similarly, from Young's inequality and Lemma 3 one can obtain

$$z_{i,n}l_{i,n}e_{i,1} \le \left(0.5l_{i,n}^2/q_{i,n}\right)z_{i,n}^2 + 0.5q_{i,n}e_i^{\mathrm{T}}e_i \tag{38}$$

$$g_{i,n} = \left(0.5l_{i,n}^2/q_{i,n}\right)z_{i,n} + \theta_i^{*\mathrm{T}}S_i\left(\hat{X}_i\right) = \varphi_{i,n}^{*\mathrm{T}}W_{i,n}\left(Z_{i,n}\right) + \Delta_{i,n},\ \left|\Delta_{i,n}\right| \le \bar{\Delta}_{i,n} \tag{39}$$

$$z_{i,n}g_{i,n} \le \frac{z_{i,n}^2}{2k_{i,n}^2}\phi_i^* W_{i,n}^{\mathrm{T}}\left(\bar{Z}_{i,n}\right)W_{i,n}\left(\bar{Z}_{i,n}\right) + \frac{1}{2}k_{i,n}^2 + \frac{1}{2}z_{i,n}^2 + \frac{1}{2}\bar{\Delta}_{i,n}^2 \tag{40}$$

$$z_{i,n}\upsilon_{k,i}^{*\mathrm{T}}\psi_{k,i} \le 0.5z_{i,n}^2 + 0.5d_{\psi k,i}\upsilon_{k,i}^{*\mathrm{T}}\upsilon_{k,i}^*, k = 1,2 \tag{41}$$

where $Z_{i,n} = [\hat{X}_i, \hat{\theta}_i, \xi_{i,n}]^{\mathrm{T}}$ and $\bar{Z}_{i,n} = [\hat{X}_i]^{\mathrm{T}}$.

Substituting (38)-(41) into (37) yields

$$\begin{aligned}\dot{V}_{i,n} \le{}& \dot{V}_{i,n-1} + z_{i,n}\left(\frac{z_{i,n}}{2k_{i,n}^2}\phi_i^* W_{i,n}^{\mathrm{T}}\left(\bar{Z}_{i,n}\right)W_{i,n}\left(\bar{Z}_{i,n}\right) + \frac{3}{2}z_{i,n}^2 + u_i - \dot{\bar{\alpha}}_{i,n} - \dot{\xi}_{i,n}\right) + \frac{q_{i,n}}{2}e_i^{\mathrm{T}}e_i \\ &- \frac{\tilde{\theta}_i^{\mathrm{T}}}{r_i}\left(r_i z_{i,n}S_i\left(\hat{X}_i\right) + \dot{\hat{\theta}}_i\right) - \frac{\tilde{\upsilon}_{1,i}^{\mathrm{T}}}{\mu_{1,i}}\left(\mu_{1,i}z_{i,n}\psi_{1,i}\left(\hat{X}_i, u_i^f\right) + \dot{\hat{\upsilon}}_{1,i}\right) + \frac{1}{2}k_{i,n}^2 + \frac{1}{2}\bar{\Delta}_{i,n}^2 \\ &- \frac{\tilde{\upsilon}_{2,i}^{\mathrm{T}}}{\mu_{2,i}}\left(\mu_{2,i}z_{i,n}\psi_{2,i}\left(u_i^f\right) + \dot{\hat{\upsilon}}_{2,i}\right) + \frac{1}{2}d_{\psi 1,i}\upsilon_{1,i}^{*\mathrm{T}}\upsilon_{1,i}^* + \frac{1}{2}d_{\psi 2,i}\upsilon_{2,i}^{*\mathrm{T}}\upsilon_{2,i}^*\end{aligned} \tag{42}$$

From (42), the adaptive laws of $\dot{\hat{\theta}}_i$, $\dot{\hat{\upsilon}}_{1,i}$, $\dot{\hat{\upsilon}}_{2,i}$ and $\dot{\hat{\phi}}_i$ are designed as follows

$$\dot{\hat{\theta}}_i = -r_i z_{i,n}S_i\left(\hat{X}_i\right) - \sigma_{1,i}\hat{\theta}_i \tag{43}$$

$$\dot{\hat{\upsilon}}_{1,i} = -\mu_{1,i}z_{i,n}\psi_{1,i}\left(\hat{X}_i, u_i^f\right) - \sigma_{2,i}\hat{\upsilon}_{1,i} \tag{44}$$

$$\dot{\hat{\upsilon}}_{2,i} = -\mu_{2,i}z_{i,n}\psi_{2,i}\left(u_i^f\right) - \sigma_{3,i}\hat{\upsilon}_{2,i} \tag{45}$$

$$\dot{\hat{\phi}}_i = \sum_{k=1}^{n}\frac{\lambda_{i,1}z_{i,k}^2}{2k_{i,k}^2}W_{i,k}^{\mathrm{T}}\left(\bar{Z}_{i,k}\right)W_{i,k}\left(\bar{Z}_{i,k}\right) - \sigma_{4,i}\hat{\phi}_i \tag{46}$$

Let $u_i = \alpha_{i,n}$. Substituting $\dot{\xi}_{i,n}$, $\alpha_{i,n}^*$ and (43)-(46) into (42) achieves

$$\begin{aligned}\dot{V}_{i,n} \le{}& \sum_{k=1}^{n}\left(-c_{i,k}z_{i,k}^2 + \kappa_{i,k}z_{i,k}\operatorname{sgn}\left(\xi_{i,k}\right) + \frac{q_{i,k}}{2}e_i^{\mathrm{T}}e_i - h_{i,k}z_{i,k}^{2p}\right) + \sum_{k}^{n-1}D_{i,k} + \frac{q_{i,0}}{2}\sum_{j\in N_i}e_j^{\mathrm{T}}e_j + \frac{\sigma_{1,i}}{r_i}\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i \\ &+ \frac{\sigma_{2,i}}{\mu_{1,i}}\tilde{\upsilon}_{1,i}^{\mathrm{T}}\hat{\upsilon}_{1,i} + \frac{\sigma_{3,i}}{\mu_{2,i}}\tilde{\upsilon}_{2,i}^{\mathrm{T}}\hat{\upsilon}_{2,i} + \frac{\sigma_{4,i}}{\lambda_{i,1}}\tilde{\phi}_i\hat{\phi}_i + \frac{k_{i,n}^2}{2} + \frac{\bar{\Delta}_{i,n}^2}{2} + \frac{d_{\psi 1,i}\upsilon_{1,i}^{*\mathrm{T}}\upsilon_{1,i}^*}{2} + \frac{d_{\psi 2,i}\upsilon_{2,i}^{*\mathrm{T}}\upsilon_{2,i}^*}{2} + w_{i,n}\end{aligned} \tag{47}$$

Using Young's inequality for further simplification as

$$\kappa_{i,k}z_{i,k}\operatorname{sgn}\left(\xi_{i,k}\right) \le \frac{1}{2}z_{i,k}^2 + \frac{1}{2}\kappa_{i,k}^2 \tag{48}$$

$$\frac{\sigma_{1,i}}{r_i}\tilde{\theta}_i^{\mathrm{T}}\hat{\theta}_i = \frac{\sigma_{1,i}}{r_i}\tilde{\theta}_i^{\mathrm{T}}\left(\theta_i^* - \tilde{\theta}_i\right) \le \frac{\sigma_{1,i}}{2r_i}\theta_i^{*\mathrm{T}}\theta_i^* - \frac{\sigma_{1,i}}{2r_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i \tag{49}$$

Generalizing (49) to other RBFNN weight terms. Then, (47) can be simplified as

$$\begin{aligned}\dot{V}_{i,n} \le{}& -\sum_{k=1}^{n}\left(\left(c_{i,k} - \frac{1}{2}\right)z_{i,k}^2 - \frac{q_{i,k}}{2}e_i^{\mathrm{T}}e_i + h_{i,k}z_{i,k}^{2p} - D_{i,k}\right) + \frac{q_{i,0}}{2}\sum_{j\in N_i}e_j^{\mathrm{T}}e_j \\ &- \frac{\sigma_{1,i}}{2r_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i - \frac{\sigma_{2,i}}{2\mu_{1,i}}\tilde{\upsilon}_{1,i}^{\mathrm{T}}\tilde{\upsilon}_{1,i} - \frac{\sigma_{3,i}}{2\mu_{2,i}}\tilde{\upsilon}_{2,i}^{\mathrm{T}}\tilde{\upsilon}_{2,i} - \frac{\sigma_{4,i}}{2\lambda_{i,1}}\tilde{\phi}_i^2\end{aligned} \tag{50}$$

where

$$D_{i,n}=\sum_{k=1}^{n}\frac{1}{2}\kappa_{i,k}^{2}+\frac{1}{2}k_{i,n}^{2}+\frac{1}{2}\bar{\Delta}_{i,n}^{2}+w_{i,n}+\frac{1}{2}d_{\psi 1,i}\upsilon_{1,i}^{*\mathrm{T}}\upsilon_{1,i}^{*}+\frac{1}{2}d_{\psi 2,i}\upsilon_{2,i}^{*\mathrm{T}}\upsilon_{2,i}^{*}$$
$$+\frac{\sigma_{1,i}}{2r_i}\theta_i^{*\mathrm{T}}\theta_i^{*}+\frac{\sigma_{2,i}}{2\mu_{1,i}}\upsilon_{1,i}^{*\mathrm{T}}\upsilon_{1,i}^{*}+\frac{\sigma_{3,i}}{2\mu_{2,i}}\upsilon_{2,i}^{*\mathrm{T}}\upsilon_{2,i}^{*}+\frac{\sigma_{4,i}}{2\lambda_{i,1}}\phi_i^{*2}$$

4 Stability analysis

Theorem 1: For the nonlinear MASs (1), under certain assumptions, if for given matrix B_i, and parameters $\tau_i>0$, $\bar{q}_i>0$, there exist observer gain matrix L_i and symmetric positive matrix P_i such that

$$A_i^{\mathrm{T}}P_i+P_iA_i+2\tau_iP_i+\left(\bar{q}_i/2\right)I+P_iB_i+B_i^{\mathrm{T}}P\le 0 \tag{51}$$

holds. Then, by control laws (21) and corresponding adaptive laws, all signals in the closed-loop system are SGUUB in finite time. Moreover, by selecting appropriate design parameters, the consensus errors can converge to a neighborhood of the origin.

Proof: Construct the Lyapunov function candidate as

$$V=\sum\nolimits_{i=1}^{N}\left(V_{e,i}+V_{i,n}\right) \tag{52}$$

From (13)(16)(50), one obtains

$$\begin{aligned}\dot{V}\le&\sum_{i=1}^{N}e_i^{\mathrm{T}}\left(A_i^{\mathrm{T}}P_i+P_iA_i+2\tau_iP_i+P_iB_i+B_i^{\mathrm{T}}P_i\right)e_i+\sum_{i=1}^{N}\Big(-\frac{\eta_{1,i}}{2r_i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i-\frac{\eta_{2,i}}{2\mu_{1,i}}\tilde{\upsilon}_{1,i}^{\mathrm{T}}\tilde{\upsilon}_{1,i}\\&-\frac{\eta_{3,i}}{2\mu_{2,i}}\tilde{\upsilon}_{2,i}^{\mathrm{T}}\tilde{\upsilon}_{2,i}-\frac{\sigma_{4,i}}{2\lambda_{i,1}}\tilde{\phi}_i^{2}+\frac{q_{i,0}}{2}\sum_{j\in N_i}e_j^{\mathrm{T}}e_j-\sum_{k=1}^{n}\left(\left(c_{i,k}-\frac{1}{2}\right)z_{i,k}^{2}-\frac{q_{i,k}}{2}e_i^{\mathrm{T}}e_i+h_{i,k}z_{i,k}^{2p}\right)\Big)+D_t\end{aligned} \tag{53}$$

where $D_t=\sum_{i=1}^{N}\left(\sum_{k=1}^{n}D_{i,k}+\|P_i\|(\bar{\delta}_i^{2}/\tau_{i,1}+\bar{\varepsilon}_{1,i}^{2}/\tau_{i,2}+\bar{\varepsilon}_{2,i}^{2}/\tau_{i,3})\right)$, $\eta_{1,i}=\sigma_{1,i}-2r_i\|P_i\|d_{S,i}/\tau_{i,1}>0$, $\eta_{2,i}=\sigma_{2,i}-2\mu_{1,i}\|P_i\|d_{\psi 1,i}/\tau_{i,2}>0$, $\eta_{3,i}=\sigma_{3,i}-2\mu_{2,i}\|P_i\|d_{\psi 2,i}/\tau_{i,3}>0$, $\bar{q}_i=\max\{q_{i,1},\ldots,q_{i,n}\}+\sum_{j\in N_i}q_{i,0}$.

Then, (53) can be further transformed into

$$\begin{aligned}\dot{V}\le&\sum_{i=1}^{N}e_i^{\mathrm{T}}\left(A_i^{\mathrm{T}}P_i+P_iA_i+2\tau_iP_i+\frac{\bar{q}_i}{2}I+P_iB_i+B_i^{\mathrm{T}}P_i\right)e_i-\sum_{i=1}^{N}\Big(\sum_{k=1}^{n}\left(h_{i,k}z_{i,k}^{2p}+(c_{i,k}-\frac{1}{2})z_{i,k}^{2}\right)+\frac{\eta_{1,i}\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i}{2r_i}\\&+\frac{\eta_{2,i}\tilde{\upsilon}_{1,i}^{\mathrm{T}}\tilde{\upsilon}_{1,i}}{2\mu_{1,i}}+\frac{\eta_{3,i}\tilde{\upsilon}_{2,i}^{\mathrm{T}}\tilde{\upsilon}_{2,i}}{2\mu_{2,i}}+\frac{\sigma_{4,i}\tilde{\phi}_i^{2}}{2\lambda_{i,1}}\Big)+D_t-\varpi\sum_{i=1}^{N}\Big(\left(e_i^{\mathrm{T}}P_ie_i\right)^{p}-\left(e_i^{\mathrm{T}}P_ie_i\right)^{p}+\eta_{1,i}\left(\frac{\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i}{2r_i}\right)^{p}-\eta_{1,i}\left(\frac{\tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i}{2r_i}\right)^{p}\\&+\eta_{2,i}\left(\frac{\tilde{\upsilon}_{1,i}^{\mathrm{T}}\tilde{\upsilon}_{1,i}}{2\mu_{1,i}}\right)^{p}-\eta_{2,i}\left(\frac{\tilde{\upsilon}_{1,i}^{\mathrm{T}}\tilde{\upsilon}_{1,i}}{2\mu_{1,i}}\right)^{p}+\eta_{3,i}\left(\frac{\tilde{\upsilon}_{2,i}^{\mathrm{T}}\tilde{\upsilon}_{2,i}}{2\mu_{2,i}}\right)^{p}-\eta_{3,i}\left(\frac{\tilde{\upsilon}_{2,i}^{\mathrm{T}}\tilde{\upsilon}_{2,i}}{2\mu_{2,i}}\right)^{p}+\sigma_{4,i}\left(\frac{\tilde{\phi}_i^{2}}{2\lambda_{i,1}}\right)^{p}-\sigma_{4,i}\left(\frac{\tilde{\phi}_i^{2}}{2\lambda_{i,1}}\right)^{p}\Big)\end{aligned} \tag{54}$$

where $0<p<1$ and ϖ are positive constants.

From (51), to stabilize system (1), there exists symmetric positive definite matrix Q_i that results in

$$A_i^{\mathrm{T}}P_i+P_iA_i+2\tau_iP_i+\frac{\bar{q}_i}{2}I+P_iB_i+B_i^{\mathrm{T}}P\le -Q_i \tag{55}$$

According to Young's inequality, there is $\left(e_i^{\mathrm{T}}P_ie_i\right)^{p}\le e_i^{\mathrm{T}}P_ie_i+\left(1-p\right)p^{p/(1-p)}$ and same for other terms, then (54) can be rewritten as

$$\dot{V}\le-\Lambda_1V-\Lambda_2V^{p}+D \tag{56}$$

where, $\underline{\lambda}_{1,i} = \min\left\{\left(\frac{\lambda_{\min}(Q_i)}{\lambda_{\max}(P_i)} - \varpi\right), 2c_{i,k} - 1, (1-\varpi)\eta_{1,i}, (1-\varpi)\eta_{2,i}, (1-\varpi)\eta_{3,i}, (1-\varpi)\sigma_{4,i}\right\}$, $\underline{\lambda}_{2,i} = \min\left\{\varpi, 2^p h_{i,k}\varpi, \eta_{1,i}\varpi, \eta_{2,i}\varpi, \eta_{3,i}\varpi, \sigma_{4,i}\varpi\right\}$, $k = 1,2,\ldots,n$, $\Lambda_1 = \Sigma_{i=1}^{N}\underline{\lambda}_{1,i}$, $\Lambda_2 = \Sigma_{i=1}^{N}\underline{\lambda}_{2,i}$, $D = D_t + \Sigma_{i=1}^{N}\left(1 + \eta_{1,i} + \eta_{2,i} + \eta_{3,i} + \sigma_{4,i}\right)(1-p)p^{p/(1-p)}$. And the value of ϖ should ensure that Λ_1 and Λ_2 are positive.

From (56) and Lemma 2, it can be derived that $V_{i,\ell}$ will converge to the set $|z_{i,\ell}| \le \min\left\{\sqrt{2D/(1-\pi)\Lambda_1}, \sqrt{(2D/(1-\pi)\Lambda_2)^{1/p}}\right\}$ in finite time T_1, where $0 < \pi < 1$. And $T_1 \le \max\left\{t_0 + \frac{1}{\pi\Lambda_1(1-p)}\ln\frac{\pi\Lambda_1 V^{1-p}(t_0) + \Lambda_2}{\Lambda_2}, t_0 + \frac{1}{\Lambda_1(1-p)}\ln\frac{\Lambda_1 V^{1-p}(t_0) + \pi\Lambda_2}{\pi\Lambda_2}\right\}$.

Furthermore, from (19), if $\xi_{i,\ell}$ is bounded in finite time, then $v_{i,\ell}$ will converge in finite time. Therefore, construct a Lyapunov function as

$$V_\xi = \sum_{i=1}^{N}\sum_{k=1}^{n}\xi_{i,k}^2 / 2 \tag{57}$$

From [22], $\left|\bar{\alpha}_{i,k+1} - \alpha_{i,k}\right| \le \varpi_{i,k}$ can be achieved in a finite time $T_{i,k2}$. Therefore, for $t \ge T_2 = \max\{T_{i,k2}\}$, by choosing $\varpi_{i,n} = \min\{\varpi_{i,k}, k = 1,\ldots,n-1\}$, $K_{i,0} = \min\{\kappa_{i,k}, k = 1,\ldots,n\}$, $\varpi_{i,0} = \max\{(d_i + \mu_i)\varpi_{i,1}, \varpi_{i,2},\ldots,\varpi_{i,n}\}$, $\Lambda_{\xi 1} = \min\{2c_{i,k} + 1\}$, $\Lambda_{\xi 2} = \sqrt{2}\min\{K_{i,0} - \varpi_{i,0}\}$,one can obtain

$$\dot{V}_\xi \le -\Lambda_{\xi 1}V_\xi - \Lambda_{\xi 2}V_\xi^{1/2} \tag{58}$$

Thus $\xi_{i,\ell}$ converges to the origin in finite time

$$T_3 \le T_2 + \frac{2}{\Lambda_{\xi 1}}\ln\frac{\Lambda_{\xi 1}V_\xi^{1/2}(T_2) + \Lambda_{\xi 2}}{\Lambda_{\xi 2}}$$

Therefore, for $t \ge \max\{T_1, T_3\}$, $|v_{i,\ell}| \le \min\left\{\sqrt{2D/(1-\pi)\Lambda_1}, \sqrt{(2D/(1-\pi)\Lambda_2)^{1/p}}\right\}$, which implies that all signals of the closed-loop system are SGUUB in finite time. The proof is completed.

5 Simulation results

Consider a MAS consisting of one leader and four followers under a directed graph in Fig.1, where the specific form is as follows

$$\begin{aligned} \dot{x}_{i,1} &= x_{i,2} \\ \dot{x}_{i,2} &= \rho_i(t)u_i + 4\tanh(0.25x_{i,1}x_{i,2}) + \beta_i(t - T_0)h_i(x_i, u_{ci}) + \Delta_i(t) \\ y_i &= x_{i,1} \end{aligned}$$

where $\rho_1(t) = 0.8 + 0.4\sin(t)$, $\rho_2(t) = 1 + 0.3\cos(t)$, $\rho_3(t) = 0.6 + 0.5\sin(2t)^2$, $\rho_4(t) = 0.9 + 0.3\cos(3t)$, $\Delta_1(t) = 0.1\sin(t)$, $\Delta_2(t) = 0.2\sin(t)^2$, $\Delta_3(t) = 0.05\sin(t)$, $\Delta_4(t) = 0.15\sin(t)$. The reference signal is $y_r = \sin(t)$.

By choosing $\bar{q}_i = 0.1$ and $\tau_i = 0.5$ to solve LMIs (51), one can obtain

$$P_i = \begin{bmatrix} 15.72 & -1.88 \\ -1.88 & 0.26 \end{bmatrix}, \quad L_i = \begin{bmatrix} 45.77 \\ 358.51 \end{bmatrix}$$

Choose the fault function $h_i(x_i, u_i) = 2 + 1.5(x_{i,1}x_{i,2} + 0.5\cos(u_i))$. The fault evolution rate and fault occurrence time are $r_{a,i} = 50$ and $T_0 = 5$, respectively.

Applying the design scheme in Theorem 1, we select the design parameters as $p = 0.99$, $c_{1,1} = c_{2,1} = 30$, $c_{3,1} = c_{4,1} = 40$, $c_{1,2} = c_{2,2} = 15$, $c_{3,2} = c_{4,2} = 20$, $\kappa_{i,1} = \kappa_{i,2} = 0.1$, $k_{i,1} = k_{i,2} = 10$, $h_{i,1} = h_{i,2} = 0.5$, $w_{i,1} = w_{i,2} = 0.01$, $\sigma_{1,i} = 0.01$, $\sigma_{2,i} = \sigma_{3,i} = \sigma_{4,i} = 10$, $\lambda_{i,1} = 1$, $\omega_{12} = \omega_{22} = 0.06$, $\omega_{32} = \omega_{42} = 0.08$, $r_i = 1/260$, $\mu_{1,i} = \mu_{2,i} = 10$ for $i = 1,2,3,4$. The initial conditions are $x_{i,1}(0) = x_{i,2}(0) = 0.5$, $\hat{x}_{i,1}(0) = \hat{x}_{i,2}(0) = 0.1$. The BLPF is chosen as $H_i^L(s) = 1/(s^2 + 1.414s + 1)$.

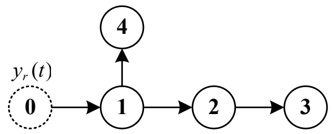

Fig. 1. Communication topology.

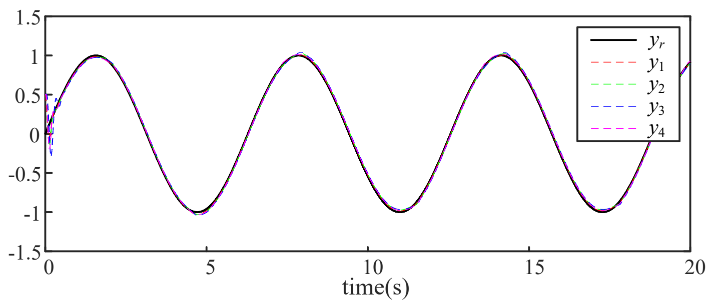

Fig. 2. System outputs y_i and reference signal y_r.

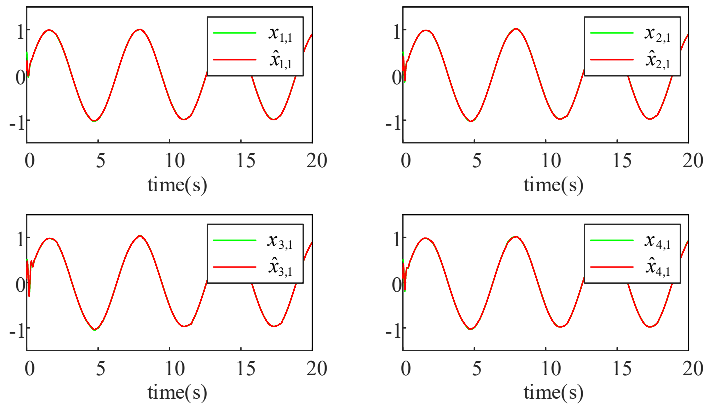

Fig. 3. Observer outputs and real signals.

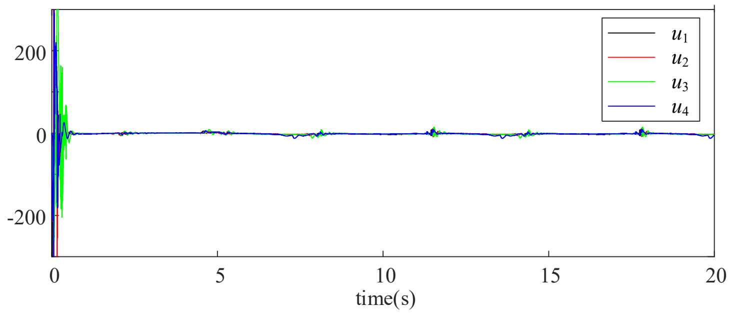

Fig. 4. Control input curves of u_i.

Fig. 2-4 display the simulation results. From these figures above, it is clearly seen that the observer is able to estimate the state quickly and efficiently. And under the action of the FTC controller designed in this paper, the system output follows the desired signal well with and without the faults.

6 Conclusion

This paper has investigated a finite time FTC scheme to achieve consensus tracking for nonlinear MASs subject to non-affine faults, unmeasurable states, uncertain control coefficients and external disturbances. The command filter and BLPF are introduced to solve “explosion of complexity” and algebraic ring problem. Furthermore, The LMI is utilized to find feasible solutions of the observer gain effectively. Based on the finite-time stability criterion and Lyapunov stability theorem, the designed observer-based finite-time controller can achieve a good tracking performance and guarantee the finite-time boundedness of all the closed-loop signals. The utilization of such control protocol in formation control problem will be considered in our future research.

References

[1] Olfati-Saber R, Murray R M. Consensus problems in networks of agents with switching topology and time-delays[J]. IEEE Transactions on Automatic Control, 2004, 49(9): 1520-1533.

[2] Ren W, Beard R W. Consensus seeking in multiagent systems under dynamically changing interaction topologies[J]. IEEE Transactions on Automatic Control, 2005, 50(5): 655-661.

[3] Su S, Lin Z. Distributed consensus control of multi-agent systems with higher order agent dynamics and dynamically changing directed interaction topologies[J]. IEEE Transactions on Automatic Control, 2016, 61(2): 515-519.

[4] Cheng L, Wang Y, Ren W, et al. On convergence rate of leader-following consensus of linear multi-agent systems with communication noises[J]. IEEE Transactions on Automatic Control, 2016, 61(11): 3586-3592.

[5] Zhao X, Chen S, Zhang Z, et al. Consensus tracking for high-order uncertain nonlinear MASs via adaptive backstepping approach[J]. IEEE Transactions on Cybernetics, 2021, 53(2): 1248-1259.
[6] Shang Y, Chen B, Lin C. Consensus tracking control for distributed nonlinear multiagent systems via adaptive neural backstepping approach[J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2018, 50(7): 2436-2444.
[7] Wang X, Li S, Wang G. Distributed optimization for disturbed second-order multiagent systems based on active antidisturbance control[J]. IEEE Transactions on Neural Networks and Learning Systems, 2019, 31(6): 2104-2117.
[8] Wang P, Wen G, Yu W, et al. Asymptotical Consensus of MIMO Linear Multiagent Systems with a Nonautonomous Leader and Directed Switching Topology: A Continuous Approach[J]. IEEE Transactions on Control of Network Systems, 2022, 10(3): 1326-1337.
[9] Yu S, Yu X, Shirinzadeh B, et al. Continuous finite-time control for robotic manipulators with terminal sliding mode[J]. Automatica, 2005, 41(11): 1957-1964.
[10] Yu J, Shi P, Zhao L. Finite-time command filtered backstepping control for a class of nonlinear systems[J]. Automatica, 2018, 92: 173-180.
[11] Zhang H, Yue D, Yin X, et al. Finite-time distributed event-triggered consensus control for multi-agent systems[J]. Information Sciences, 2016, 339: 132-142.
[12] Zhao L, Yu J, Lin C, et al. Adaptive neural consensus tracking for nonlinear multiagent systems using finite-time command filtered backstepping[J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2017, 48(11): 2003-2012.
[13] Zhao X, Chen S, Zhang Z, et al. Observer-based adaptive scaled tracking control for nonlinear MASs via command-filtered backstepping[J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2022, 53(1): 425-437.
[14] Chen B, Zhang H, Liu X, et al. Neural observer and adaptive neural control design for a class of nonlinear systems[J]. IEEE Transactions on Neural Networks and Learning Systems, 2017, 29(9): 4261-4271.
[15] Zheng X, Li X M, Yao D, et al. Observer-based finite-time consensus control for multiagent systems with nonlinear faults[J]. Information Sciences, 2023, 621: 183-199.
[16] Long J, Wang W, Huang J, et al. Distributed adaptive control for asymptotically consensus tracking of uncertain nonlinear systems with intermittent actuator faults and directed communication topology[J]. IEEE Transactions on Cybernetics, 2019, 51(8): 4050-4061.
[17] Qin J, Zhang G, Zheng W X, et al. Adaptive sliding mode consensus tracking for second-order nonlinear multiagent systems with actuator faults[J]. IEEE Transactions on Cybernetics, 2018, 49(5): 1605-1615.
[18] Tong S, Zhou H, Li Y. Neural network event-triggered formation fault-tolerant control for nonlinear multiagent systems with actuator faults[J]. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 2023.
[19] Li Y X, Yang G H. Fuzzy adaptive output feedback fault-tolerant tracking control of a class of uncertain nonlinear systems with nonaffine nonlinear faults[J]. IEEE Transactions on Fuzzy Systems, 2015, 24(1): 223-234.
[20] Cheng S, Xin B, Wang Q, et al. Dynamic switching event-triggered fixed-time cooperative control for nonlinear multi-agent systems subject to non-affine faults[J]. Nonlinear Dynamics, 2024, 112(2): 1087-1103.
[21] Zou A M, Hou Z G, Tan M. Adaptive control of a class of nonlinear pure-feedback systems using fuzzy backstepping approach[J]. IEEE Transactions on Fuzzy Systems, 2008, 16(4): 886-897.
[22] Sun J, He H, Yi J, et al. Finite-time command-filtered composite adaptive neural control of uncertain nonlinear systems[J]. IEEE Transactions on Cybernetics, 2020, 52(7): 6809-6821.